Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Wednesday 15 January 2020

Morning (Time: 2 hours 30 minutes) | Paper Reference **WMA02/01**

Mathematics

International Advanced Level
Core Mathematics C34

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. $$f(x) = 2x^4 + x^2 - 3x + 8$$

The curve with equation $y = f(x)$ has a single turning point when $x = \alpha$

(a) Show that α is a solution of the equation

$$x = \sqrt[3]{\frac{3 - 2x}{8}}$$

(3)

The iterative formula

$$x_{n+1} = \sqrt[3]{\frac{3 - 2x_n}{8}} \qquad x_1 = 0.6$$

is used to find an approximate value for α.

(b) Calculate the value of x_2 and the value of x_3 giving your answers to 4 decimal places.
(2)

(c) By choosing a suitable interval, show that $\alpha = 0.607$ to 3 decimal places.
(2)

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Question 1 continued

Q1

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2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(\frac{1}{4} - 3x\right)^{\frac{1}{2}}$$

giving each coefficient in its simplest form.

(5)

By substituting $x = \frac{1}{100}$ into the answer for (a),

(b) find an approximation for $\sqrt{22}$, giving your answer to 4 decimal places.

(2)

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Question 2 continued

(Total 7 marks)

Q2

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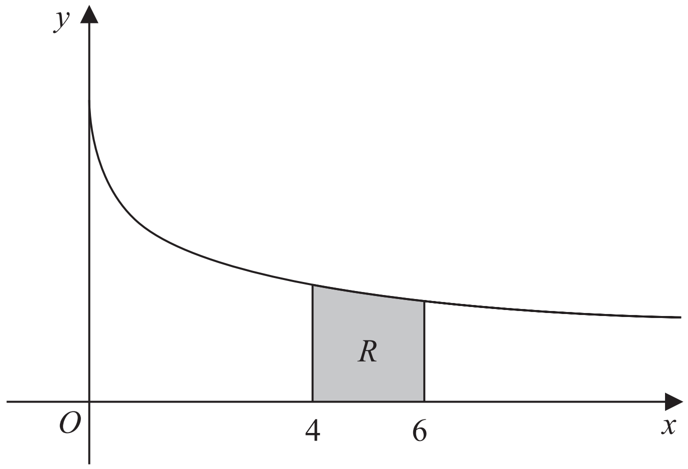

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{10}{1+\sqrt{x}}, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 6$

(a) Use the trapezium rule, with four strips, to find an estimate for the area of R. Show your working and give your answer to 2 decimal places.

(4)

(b) Using your answer to part (a) and making your method clear, estimate the value of

(i) $\displaystyle\int_2^3 \frac{60}{1+\sqrt{2x}}\,dx$

(ii) $\displaystyle\int_4^6 \frac{13+3\sqrt{x}}{1+\sqrt{x}}\,dx$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

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Question 3 continued

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Question 3 continued

Q3

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4. (a) Sketch the graph with equation

$$y = |2x - 7| + 1$$

stating the coordinates of the minimum point and the coordinates of the point where the graph cuts the y-axis.

(3)

(b) Solve the equation

$$14 - x = |2x - 7| + 1$$

(4)

A straight line l has equation $y = \frac{1}{2}x + k$, where k is a constant.

Given that l does not meet or intersect $y = |2x - 7| + 1$

(c) find the range of possible values of k.

(2)

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Question 4 continued

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Question 4 continued

Q4

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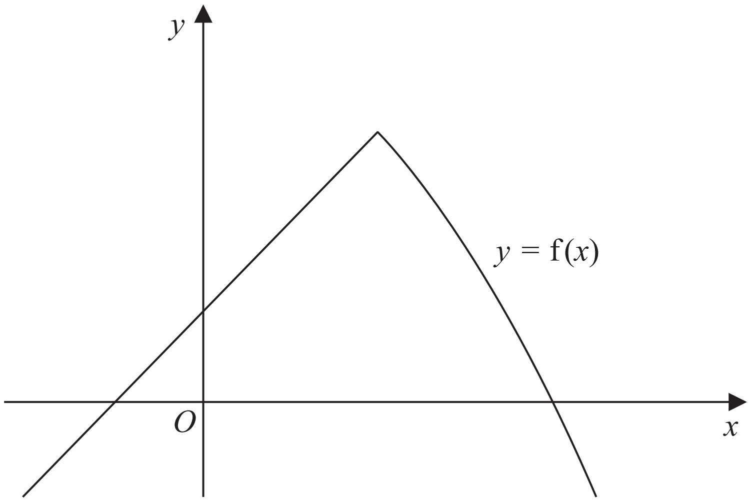

Figure 2

The continuous function f is defined by

$$\mathrm{f}(x) = \begin{cases} 9 + 3x & x \leqslant 6 \\ B - Ax^2 & x > 6 \end{cases} \quad \text{where } A \text{ and } B \text{ are positive constants}$$

A sketch of $y = \mathrm{f}(x)$ is shown in Figure 2.

(a) Find the range of f.

(1)

Given that one of the solutions of the equation $\mathrm{f}(x) = 0$ is 12

(b) (i) find the other solution,

(ii) find the value of A and the value of B.

(4)

(c) Find the value of $\mathrm{ff}(0)$.

(2)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 7 marks)

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6. Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x \ln x}{y} \qquad x > 0, \quad y > 0$$

and $y = 4$ when $x = 1$, find the exact value of y when $x = \mathrm{e}$.

(7)

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Question 6 continued

Q6

(Total 7 marks)

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7. Given $y = 3x(2x - 5)^4$

(a) show that

$$\frac{dy}{dx} = 15(2x - 5)^3(Ax + B)$$

where A and B are integers to be found.

(4)

(b) Hence find the range of values of x for which y is decreasing.

(2)

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Question 7 continued

Q7

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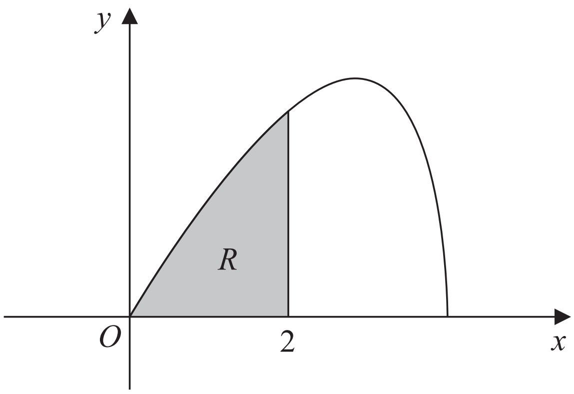

Figure 3

Figure 3 shows a sketch of the curve with parametric equations

$$x = 4\sin t, \quad y = 3\sin 2t, \quad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = 2$

Find the exact area of R.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(7)

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Question 8 continued

Q8

(Total 7 marks)

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9. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \quad \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$l_2 : \quad \mathbf{r} = \begin{pmatrix} 10 \\ c \\ 3 \end{pmatrix} + \mu \begin{pmatrix} a \\ b \\ -2 \end{pmatrix}$$

where a, b and c are constants and λ and μ are scalar parameters.

Given that

- l_1 and l_2 meet when $\lambda = -2$
- l_1 and l_2 are perpendicular

find the value of a, the value of b and the value of c.

(6)

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Question 9 continued

Q9

(Total 6 marks)

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10. The curve C has equation

$$y^3 + 4x^2y - 2x = 0$$

(a) Find, in simplest form, $\dfrac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence find the coordinates of the points on C where $\dfrac{dy}{dx} = 0$

(6)

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Question 10 continued

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Question 10 continued

Q10

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11. (a) Prove

$$\frac{\cos 3\theta}{2\sin\theta} + \frac{\sin 3\theta}{2\cos\theta} \equiv \cot 2\theta \qquad \theta \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 < x < \frac{\pi}{2}$

$$\frac{\cos 3x}{2\sin x} + \frac{\sin 3x}{2\cos x} = 5\cos 2x$$

giving your answers to 3 decimal places where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

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Question 11 continued

Q11

(Total 8 marks)

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12. Find, showing all steps of your working, the exact value of

$$\int_1^3 \frac{3x^2 + 8}{x^2 - 4x} \, \mathrm{d}x$$

(7)

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Q12

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13. A curve has parametric equations

$$x = t^2 + 3t \qquad y = \frac{2t}{1-t} \qquad t \neq 1$$

(a) Find $\frac{dy}{dx}$ in terms of t, giving your answer as a simplified fraction.

(4)

(b) Find an equation for the tangent to the curve at the point P, where $t = 2$
Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

The tangent to the curve at P cuts the curve at the point Q.

(c) Use algebra to find the coordinates of Q.

(5)

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Question 13 continued

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Question 13 continued

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Question 13 continued

Q13

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14. A scientist is studying a population of lizards on an island.

The number of lizards, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{1800}{2 + 3\mathrm{e}^{-0.2t}} \qquad t \in \mathbb{R}, t \geqslant 0$$

Use the model to answer parts (a), (b), (c) and (d).

(a) Find the number of lizards in the population at the start of the study.

(1)

The model predicts an upper limit to the number of lizards on the island.

(b) State the value of this limit.

(1)

(c) Find the value of t when $N = 780$. Give your answer to one decimal place.

(4)

(d) (i) Show that the rate of growth, $\frac{\mathrm{d}N}{\mathrm{d}t}$, is given by

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(900 - N)}{A}$$

where A is a constant to be found.

(ii) Hence state the value of N at which the rate of growth is a maximum.

(5)

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Question 14 continued

Q14

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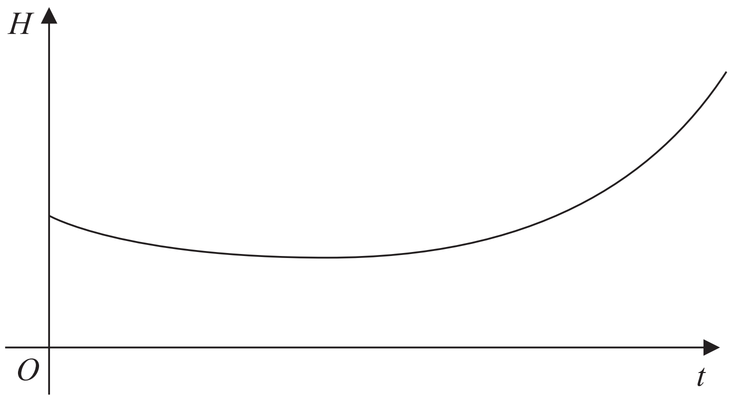

Figure 4

A jet is flying over the sea.

The height above sea level, H metres, of the jet is modelled by the equation

$$H = \frac{8000}{56 + 9\cos t° + 40\sin t°} \qquad 0 \leqslant t \leqslant 200$$

where t is the time in seconds, measured from when the jet passed over a boat.

Figure 4 is a sketch showing the graph of H against t.

Use the model to answer parts (a), (c) and (d).

(a) Find the height above sea level of the jet as it passed over the boat.

(1)

(b) Write $9\cos t° + 40\sin t°$ in the form $R\cos(t - \alpha)°$ where $R > 0$ and $0 < \alpha < 90$

Give the exact value of R and the value of α to one decimal place.

(3)

(c) Find (i) the minimum height of the jet above sea level,

(ii) the value of t at which this minimum height occurs.

(3)

(d) Find the value of t when the jet is 150 m above sea level.

(4)

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Q15

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TOTAL FOR PAPER: 125 MARKS

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