Mark Scheme (Provisional)

Summer 2021

Pearson Edexcel International Advanced Level In Mechanics M2 (WME02/01)

General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.

- **B** marks are unconditional accuracy marks (independent of M marks)

- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt

   - ft – follow through

   - the symbol $\sqrt{}$ will be used for correct ft

   - cao – correct answer only

   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark

   - isw – ignore subsequent working

   - awrt – answers which round to

   - SC: special case

   - oe – or equivalent (and appropriate)

   - d… or dep – dependent

   - indep – independent

   - dp decimal places

   - sf significant figures

   - ✳ The answer is printed on the paper or ag- answer given

   - ⌐ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking
(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A)   Taking moments about A.
  N2L   Newton's Second Law (Equation of Motion)
  NEL   Newton's Experimental Law (Newton's Law of Impact)
  HL   Hooke's Law
  SHM   Simple harmonic motion
  PCLM   Principle of conservation of linear momentum
  RHS, LHS   Right hand side, left hand side.

Q	Solution	Mark	Notes
1	Driving force $(F) = \dfrac{3500}{V}$	B1	Use of $P = Fv$
	Equation of motion: $F - 20V + 480g\sin\theta = 0$	M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion
	$\dfrac{3500}{V} - 20V + 40g = 0$	A1	Correct unsimplified equation in V.
	$20V^2 - 392V - 3500 = 0$	M1	Form a 3 term quadratic equation $(=0)$ in V
	$V = 26.3$ (26)	A1	3 sf or 2 sf Not $\dfrac{49 + 22\sqrt{14}}{5}$ (follows use of 9.8)
		(5)	
		[5]	

2a			Allow column vectors throughout
	Use $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$	M1	Differentiate – at least 3 powers going down by 1
	$\mathbf{a} = \left(10t - 3t^2\right)\mathbf{i} + \left(6t^2 - 8\right)\mathbf{j}$	A1	
	$\mathbf{F} = 1.5 \times \left(\left(20 - 12\right)\mathbf{i} + \left(24 - 8\right)\mathbf{j}\right)$	DM1	Substitute $t = 2$ and use $\mathbf{F} = m\mathbf{a}$ Dependent on preceding M1
	$= 12\mathbf{i} + 24\mathbf{j}$	A1	Ignore magnitude of $\mathbf{F}$ if found
		(4)	
2b	$5t^2 - t^3 = 0 \Rightarrow t = 5$	B1	(Not moving when $t = 0$ so no need to mention $t = 0$)
	Use of $\mathbf{r} = \displaystyle\int \mathbf{v}\,dt$	M1	Integrate to find $\mathbf{r}$ – at least 3 powers going up by 1.
	$\mathbf{r} = \left(\dfrac{5}{3}t^3 - \dfrac{1}{4}t^4\right)\mathbf{i} + \left(\dfrac{1}{2}t^4 - 4t^2\right)\mathbf{j}$	A1	Condone if no constant of integration seen (since $t = 0, \mathbf{r} = \mathbf{0}$)
	$\mathbf{r} = \left(\dfrac{625}{12}\right)\mathbf{i} + \left(\dfrac{425}{2}\right)\mathbf{j}$	A1	Final answer $52\mathbf{i} + 210\mathbf{j}$ or better $\left(52.08\dot{3}\mathbf{i} + 212.5\mathbf{j}\right)$
		(4)	
		[8]	

3a		square	triangle	circle	T		
	mass	36	8	π	$28-\pi$		
	c from AD	$3a$	$\dfrac{7}{3}a$	$4a$	d		
	Mass ratio					B1	
	Distances from AD or a parallel axis					B1	
	M(AD or parallel axis):					M1	Moments equation. Need all terms and dimensionally correct. Condone sign errors.
	$36 \times 3a - 8 \times \dfrac{7}{3}a - \pi \times 4a = (28-\pi)d$					A1	Correct unsimplified equation for their parallel axis
	$\left(108a - \dfrac{56}{3}a - 4\pi a = (28-\pi)d\right)$						
	$d = \dfrac{324 - 56 - 12\pi}{3(28-\pi)}a = \dfrac{4(67-3\pi)}{3(28-\pi)}a$ *					A1*	Obtain **given answer** from correct working
							Distance from BC is $\dfrac{(236-6\pi)a}{3(28-\pi)}$ Allow 4/5 if seen.
						(5)	
3b	M(A):					M1	Complete method to form an equation in k and W only Dimensionally correct.
	$W \times \dfrac{4(67-3\pi)}{3(28-\pi)}a = kW \times 6a$					A1	Correct unsimplified equation
	$k = 0.51$					A1	Q asks for 2dp
						(3)	
						[8]	

4			Resolving parallel and perpendicular to the original direction of motion
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ parallel or perpendicular to original direction
	$J\cos 30° = 2.4\cos\theta$ or $\quad J\cos 60° = 2.4\sin\theta - 1.5$	A1	One correct unsimplified equation
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ to form second equation
		A1	2nd correct unsimplified equation
	The first 4 marks are available for a correct equation in vector form.		$\begin{pmatrix} -2.4\cos\theta \\ 2.4\sin\theta \end{pmatrix} = \begin{pmatrix} -J\cos 30° \\ J\cos 60° + 1.5 \end{pmatrix}$
	$2.4^2 = \dfrac{3J^2}{4} + \dfrac{J^2}{4} + 1.5J + 1.5^2$ $\left(J^2 + 1.5J - 3.51 = 0 \right)$	DM1	Form an equation in J only Dependent on previous two M1 marks
	$\quad J = 1.3$	A1	1.3 or better (1.268….)
		(6)	
			See over for alternatives

4 Alt 1			Resolving parallel and perpendicular to the direction of the impulse.
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ in any direction
	$J = 0.3(8\cos\alpha - 5\cos 60°)$ Or $5\sin 60° = 8\sin\alpha$	A1	Correct unsimplified equation $\left(\begin{array}{c}2.4\cos\alpha = J + 1.5\cos 60° \\ 2.4\sin\alpha = 1.5\sin 60°\end{array}\right)$
	Use of $J = m(v-u)$	M1	Use of $J = m(v-u)$ in perpendicular direction
		A1	Correct unsimplified equation
	$2.4^2 = \left(J + \dfrac{3}{4}\right)^2 + \left(\dfrac{3}{2}\right)^2 \times \dfrac{3}{4}$ $\left(J^2 + 1.5J - 3.51 = 0\right)$	DM1	Form an equation in J only Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268….)
	Could have a mixture of the first 2 alternatives. M1A1M1A1 for 2 independent equations. DM1A1 for solving		
		(6)	
4 Alt 2			Using vector triangle.
	Impulse momentum triangle	M1	Form dimensionally correct vector triangle (for impulse or momentum)
	Use of cosine rule	M1	Use of cosine rule in momentum or velocity triangle
	$2.4^2 = J^2 + 1.5^2 - 3J\cos 120°$	A1	unsimplified equation in v or mv with at most one error
		A1	Correct unsimplified equation
	$J^2 + 1.5J - 3.51 = 0$	DM1	Form a simplified equation in J Dependent on previous two M1 marks
	$J = 1.3$	A1	1.3 or better (1.268….)
		(6)	
		[6]	

5a			
	Moments about A:	M1	Need all terms and dimensionally correct. Condone sign errors and sin/cos confusion Or complete method to form equation in T (and M).
	$5a \times T \sin 55° = 4a \cos 20° \times Mg$	A1	Correct unsimplified equation in T (and M).
	$T = \dfrac{4 \cos 20°}{5 \sin 55°} Mg \left(= 0.918 Mg\right)$	A1	Or equivalent (Exact or $0.92Mg$ or better)
		(3)	
5b	Resolve vertically	M1	Need all terms. Condone sign errors and sin/cos confusion
	$\updownarrow : Mg = V + T \cos 55°$ $\left(V = 0.47...Mg\right)$	A1	Correct unsimplified equation in T or their T
	Resolve horizontally	M1	Condone consistent sin/cos confusion
	$H = T \sin 55°$ $\left(H = 0.75...Mg\right)$	A1	Correct unsimplified equation in T or their T
	Resultant $\lambda = \sqrt{\left(0.4736..\right)^2 + \left(0.7517..\right)^2}$	M1	Substitute for T and use Pythagoras
	$= 0.89$	A1	The Q asks for 2 sf
		(6)	
5balt	Moments about B	M1	Dimensionally correct. Need all terms. Condone sign errors and sin/cos confusion
	$Mga \cos 20° + 5aH \cos 70° = 5aV \cos 20°$	A1	Correct unsimplified equation
	Moments about C	M1	Dimensionally correct. Condone sign errors and sin/cos confusion
	$5aH = 4aMg \cos 20°$	A1	Correct unsimplified equation
	Resultant $\lambda = \sqrt{\left(0.4736..\right)^2 + \left(0.7517..\right)^2}$	M1	Use Pythagoras
	$= 0.89$	A1	The Q asks for 2 sf

		M1A1M1A1 for 2 independent equations M1A1 to solve for λ		
6a	GPE lost		M1	Need all terms. Condone sign errors and sin/cos confusion
	$= 3g \times 2 - 2g \times 2\sin\theta$ $\left(= 6g - 4g \times \dfrac{5}{13}\right)$		A1	Correct unsimplified. Accept $\pm$
	$= \dfrac{58}{13}g = 43.7\,(44)\,(\text{J})$		A1	Must be positive. Exact multiple of g or 3 sf or 2 sf
			(3)	
6b	Normal reaction $= 2g\cos\theta\left(=\dfrac{24}{13}g\right)$		B1	
	$F_{\max} = \dfrac{3}{8} \times R\left(=\dfrac{9g}{13}\right)$		M1	Use $F = \mu R$ with their R
	Work done $= 2 \times F_{\max}$		M1	Their $F_{\max}$
	$\left(=\dfrac{18g}{13}\right) = 13.6(\text{J})\ 14\,(\text{J})$		A1	Exact multiple of g or 3 sf or 2 sf
			(4)	
6c	Total KE gained = GPE lost - total WD against friction		M1	Must be using work-energy. Dimensionally correct. Required terms and no extras. Condone sign errors.
	$\dfrac{1}{2}(2+3)v^2 = (their(a)) - (their(b))$ $\left(\dfrac{5}{2}v^2 = \dfrac{58}{13}g - \dfrac{18}{13}g = \dfrac{40}{13}g\right)$		A2ft	Follow their (a) and (b) -1 each error
	$v = \sqrt{\dfrac{16}{13}g} = 3.47\left(\text{m s}^{-1}\right)$ or $3.5\left(\text{m s}^{-1}\right)$		A1	3 sf or 2 sf (need to substitute for g)
			(4)	
6d	KE lost = GPE gained + WD against friction		M1	Must be using work-energy. Dimensionally correct. Required terms and no extras. Condone sign errors.
	$\dfrac{1}{2} \times 2 \times \dfrac{16}{13}g = 2g \times d\sin\theta + \dfrac{3}{8} \times 2g \times \dfrac{12}{13}d$ $\dfrac{1}{2} \times 2 \times v^2 = 2g \times d\sin\theta + d \times F_{\max}$ $\dfrac{16}{13}g = \left(\dfrac{10}{13}g + \dfrac{9}{13}g\right)d$		A2ft	Follow their (c) and their $F_{\max}$ -1 each error
	$d = \dfrac{16}{19}$		A1	g cancels. 0.84 or better $\left(0.8421....\right)$
			[15]	

7a	$-12 = 12 - gt$	M1	Use *suvat* to find time taken
	$t = \dfrac{24}{g}(= 2.45)$	A1	
	$AB = 6t$	M1	Horizontal distance
	$= 14.7\,(15)\,(\text{m})$	A1	3 sf or 2 sf Not $\dfrac{720}{49}$ (follows use of 9.8) Not $\dfrac{144}{g}$ (do not accept *g* in the denominator)
		(4)	
7b	Vertical component of velocity $= (\pm)8$	B1	
	$v^2 = u^2 + 2as$	M1	Complete method using *suvat* to find *h*
	$\Rightarrow 8^2 = 12^2 - 2gh$	A1	Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf Not $\dfrac{200}{49}$ (follows use of 9.8) Not $\dfrac{40}{g}$ (do not accept *g* in the denominator)
		(4)	
7b alt	$\mathbf{v} = \begin{pmatrix} 6 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ g \end{pmatrix} t \Rightarrow 12 - gt = (\pm)8$	B1	Correct expression for critical value(s) of *t*
	$h = 12t - \dfrac{1}{2}gt^2$	M1	Complete method using *suvat* to find *h*
	$= \dfrac{48}{g} - \dfrac{8}{g}$ or $= \dfrac{240}{g} - \dfrac{200}{g}$	A1	Correct unsimplified equation
	$h = 4.08 \quad (4.1)$	A1	3 sf or 2 sf
		(4)	
7c	$\begin{pmatrix} 6 \\ -12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ v \end{pmatrix} = 0$	M1	Complete method to find vertical component at *C*.
	$\Rightarrow v = 3$	A1	
	$\mathbf{v} = 6\mathbf{i} + 3\mathbf{j} \ (\text{m s}^{-1})$	A1	Must be a vector in terms of $\mathbf{i}$ and $\mathbf{j}$
	If see $\begin{pmatrix} 6 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ v \end{pmatrix} = 0$ leading to $\mathbf{v} = 6\mathbf{i} - 3\mathbf{j}$ mark as a misread: M1A0A0		
		(3)	
		[11]	
	Accept working in column vectors throughout apart from the final A1		

8a			
	Use CLM: $4mu = 2mv + mw$	M1	Need all terms. Condone sign errors. Dimensionally correct but allow with m cancelled
	$\left(4u = 2v + w\right)$	A1	Correct unsimplified. Signs correct for their v, w
	Use Impact law	M1	Used the right way round. Condone sign errors.
	$w - v = 2ue$	A1	Correct unsimplified. Signs consistent with CLM equation.
	$\Rightarrow 4u = 2\left(w - 2ue\right) + w$	DM1	Solve for v or w. Dependent on previous 2 M marks
	$3w = 4u + 4ue, \quad w = \dfrac{4}{3}u\left(1 + e\right)$ *	A1*	Obtain **given result** from correct working
	$v = \dfrac{2}{3}u\left(2 - e\right)$	A1	Or equivalent. Must be positive
		(7)	
8b	$2 > e$ so A moving towards centre	B1	Correct statement about direction of travel for A or B
	$mw - 3mu = mx + 3my$ $y - x = e\left(u + \dfrac{4u}{3} + \dfrac{4eu}{3}\right)$	M1	Use CLM and impact law correctly to form simultaneous equations in x and y.
	$\dfrac{4}{3}eu - \dfrac{5}{3}u = x + 3y$ $3y - 3x = e\left(7u + 4ue\right)$	A1	Both equations correct unsimplified
	$4x = \dfrac{4}{3}ue - \dfrac{5}{3}u - 7ue - 4ue^2$	DM1	Solve for x
	$x = -\dfrac{5}{12}u - \dfrac{17}{12}ue - ue^2$	A1	
	$e > 0, u > 0$ so B moving towards centre from opposite direction, hence they collide.*	A1*	Obtain **given answer** from correct working
		(6)	
	Alternative for last 3 marks;		
	C moving towards centre implies B moving towards centre, so collision. C moving away from centre , so $y > 0$,	DM1	Consider direction of C

	$x = w - 3u - 3y = -\dfrac{8u}{3} + \dfrac{4eu}{3} - 3y$		
	$= -\dfrac{u}{3}(8 - 4e) - 3y$	A1	
	< 0 because $e \leq 1$ and $y > 0$ hence B moving towards centre from opposite direction, and they will collide.*	A1*	Obtain **given answer** from correct working
		[13]	