Pearson Edexcel

Mark Scheme (Provisional)

Summer 2021

Pearson Edexcel International Advanced Subsidiary/Advanced Level
In Pure Mathematics P2 (WMA12/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ∟ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the ‘0’ column when it was meant to be ‘1’ and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:
1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:
1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks
1(a)	(i) There is a common difference (no common ratio) and so an arithmetic series should be used.	B1
	(ii) $(u_n)=5+(n-1)"d"$ or $(u_n)=5+n"d"$	M1
	$(u_n)=5+0.25(n-1)$ oe	A1
		(3)
(b)	Need $S_n=\frac{n}{2}\left(2\times 5+(n-1)\times 0.25\right)\geqslant 350$	M1A1
	$\Rightarrow 0.25n^2+9.75n\geqslant 700\Rightarrow (n+19.5)^2-19.5^2\geqslant 2800\Rightarrow n=...$	M1
	So 37 weeks.	A1
		(4)
		(7 marks)

Notes:

(a)(i)

B1: Identifies that there is a common difference (e.g. goes up in equal amounts) between first and second, and second and third terms, or that the ratio between first and second, and second and third is not the same, and states arithmetic series/sequence to be used.

M1: Attempts general term for A.S. with n or $n-1$ used and their d (which need not be correct).

A1: Correct expression for general term, accept equivalents, eg $4.75+0.25n$. May use another label than u_n or no label at all.

(b)

M1: Uses the sum formula for A.S. with their a and d and equates to, or sets inequality with, 350. Accept with any inequality symbol or equality between.

A1: Correct equation/inequality. Accept with any inequality symbol or equality between.

M1: Forms and solves a 3 term quadratic (need not be gathered to one side). Any valid method (including calculator – you may need to check).

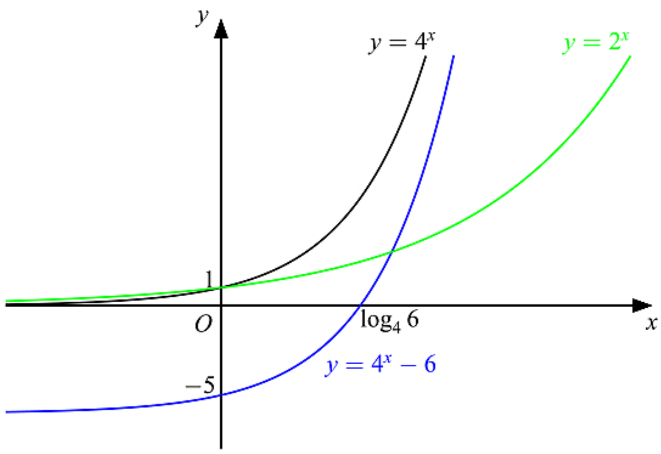
A1: 37 weeks selected as answer.

For attempts via listing, send to review.

For use of geometric series:

Case 1: If a student states arithmetic series but uses geometric series formulae, then only the B mark can be scored in (a), but in (b) allow M0A0M1A0 if a correct method is used to solve their equation $\frac{5("r"^n-1)}{"r"-1}\geqslant 350$ (or with "=" or "<" etc) to find n (so $n=\log_{"r"}(70("r"-1)+1)$ oe)

Case 2: If a student states a geometric series and gives a common ratio, then allow B0M1A0 in (a) for $(u_n)=5"r"^k$ with $k=n$ or $n-1$ and in (b) allow M1A0 for $\frac{5("r"^n-1)}{"r"-1}\geqslant 350$ and then M1A0 for a correct method to solve this equation.

Question	Scheme		Marks
2(a)		(i) Correct shape and position for $y = 2^x$ Crosses y axis with same intersection as $y = 4^x$ but with gentler slope.	B1
		(ii) Graph of same shape as $y = 4^x$ but translated down	B1
		y intercept at -5	B1
		x intercept at $\log_4 6$	B1
			(4)
(b)	$2^x = 4^x - 6$ or $y = (2^x)^2 - 6 = y^2 - 6$		M1
	$\Rightarrow 2^{2x} - 2^x - 6 = 0 \Rightarrow \left(2^x - 3\right)\left(2^x + 2\right) = 0 \Rightarrow 2^x = \ldots$ or $\left(y-3\right)\left(y+2\right) = 0 \Rightarrow y = \ldots$		M1
	$y = 2^x = 3$		A1
	$x = \log_2 3$ oe		A1
			(4)
			(8 marks)

Notes:

(a)(i)

Note: Be tolerant of "wobbles" in the graph if it is clear the correct shapes are meant, but the graph must not clearly bend away from the asymptotes.

B1: Correct sketch for $y = 2^x$. This should be correct shape, but shallower gradient – look for crossing at same y intercept as $y = 4^x$, above $y = 4^x$ to the left of the y axis and below $y = 4^x$ to the right of the y axis.

(ii)

B1: Graph of $y = 4^x$ translated down by 6 units. Look for the same shape, always below $y = 4^x$ though be tolerant as the graph increases (accept if the graphs don't "narrow" as shown).

B1: Correct y intercepts of -5

B1: Correct x intercept of $\log_4 6$. Accept equivalents such as $\log 6 / \log 4$ or awrt 1.29

Ignore any references to asymptotes for these marks.

(b)

M1: Sets the equations equal to form an equation in x only, or writes 4^x in terms of 2^x and substitutes to achieve an equation in y only.

M1: Solves the equation to find a value for 2^x or y

A1: Correct y (or 2^x) value of intersection. (May have second solution)

A1: Correct x value of intersection. Must have rejected second "solution". Accept exact equivalents, such as $\log 3 / \log 2$ or $2\log_4 3$

Question	Scheme	Marks
3(i)	At least three of: For $p = 2$: $2^3 + 2 = 8 + 2 = 10$; For $p = 3$: $3^3 + 3 = 27 + 3 = 30$ For $p = 5$: $5^3 + 5 = 125 + 5 = 130$; For $p = 7$: $7^3 + 7 = 343 + 7 = 350$	M1
	Each case gives a multiple of 10. As 2,3,5 and 7 are the only single digit primes, the result has been proved for all single digit primes.	A1
		(2)
(ii)	$(n+1)^3 - n^3 = n^3 + 3n^2 + 3n + 1 - n^3 = 3n^2 + 3n + 1$	M1A1
	$= 3(n^2 + n) + 1$ which is one more than a multiple of 3, so is not divisible by 3 for any $n \in \mathbb{N}$	A1
		(3)
		(5 marks)

Notes:

(i)
M1: Checks result for at least three of the four single digit primes (2,3,5 and 7) – attaining a multiple of 10 is enough, no need to see the product. Allow if there are slips.
A1: All four cases correctly checked, with minimal conclusion that the result is true. Ignore checks on non-prime values such as $p = 8$ which gives $8^3 + 8 = 520$, but award A0 if the case $p = 1$, leads to a conclusion that the result is not true.

(ii)
Note this appears as MMA on ePEN but is being marked as MAA.
M1: Expands to a four term cubic (may have incorrect coefficients) and then cancels the n^3 terms
A1: Correct quadratic in n.
A1: Correct explanation and conclusion given. For the explanation accept e.g factors out 3 from the relevant terms to achieve a form $3 \times (n^2 + n) + 1$ which is one more than a multiple of 3, or explains each term other than 1 is divisible by 3

Question	Scheme	Marks
4(a)	$\left(2+\frac{x}{8}\right)^{13} = 8192 + \ldots$	B1
	$+\underline{\binom{13}{1}}2^{12}\left(\frac{x}{8}\right)^{1} + \underline{\binom{13}{2}}2^{11}\left(\frac{x}{8}\right)^{2} + \underline{\binom{13}{3}}2^{10}\left(\frac{x}{8}\right)^{3} + \ldots$	M1
	$\left(2+\frac{x}{8}\right)^{13} = (8192) + 6656x + 2496x^2 + 572x^3 + \ldots$	A1A1
		(4)
(b)	$\frac{x}{8} = 0.0125 \Rightarrow x = 0.1$	B1
	$2.0125^{13} \approx 8192 + 665.6 + 24.96 + 0.572$	M1
	$= 8883.132$ cao	A1
		(3)
(c)	As all the terms in the expansion are positive, the truncated series will give an underestimate of the actual value.	B1
		(1)
		(8 marks)

Notes:

(a)

B1: For the correct constant term of 8192 in their expression. Do not accept 2^{13}

M1: Correct binomial coefficients linked with correct powers of x in their expansion. Accept alternative notation, such as ${}^{13}C_1$ or just 13 etc, for the coefficients. (The power 1 need not be seen.) The powers of 2 and the 8's may be missing or incorrect.

A1: Any two of the final three terms correct. (See also note below.)

A1: All three of the final terms correct.

Note: Students may attempt to take out a common factor of 2^{13} first. In such cases the **B** mark is for correct constant term in their final expansion, **M** follows as above, correct coefficients linked to correct powers of x. The first **A** can be awarded as above or SC $8192\left(1+\frac{13}{16}x+\frac{39}{128}x^2+\frac{143}{2048}x^3\right)$ (oe), final **A** as scheme

(b)

B1: Identifies $x = 0.1$ needs to be substituted into the expression.

M1: Substitutes their $x = 0.1$ into their answer to (a)

A1: Correct answer only, must be to 3 dp.

(c)

B1: States or implies underestimate with correct explanation of why the answer found is an underestimate. Must refer to terms in the expansion being positive or equivalent wording. Do not accept just "as adding more terms" without reference to the terms being non-negative.

If all that is done is a comparison with the true value, then B0.

Question	Scheme	Marks
5(a)	$h=\frac{1}{4}$	B1
	$A\approx\left(\frac{1}{2}\times\frac{1}{4}\right)\{5.453+5+2(7.764+...+7.626)\}$	M1
	$A\approx\left(\frac{1}{2}\times\frac{1}{4}\right)\{5.453+5+2(7.764+9.375+9.964+9.367+7.626)\}$	A1
	$=$ awrt 12.33	A1
		(4)
(b)	$\int x^{\frac{3}{2}}-3x+9\,dx=kx^{\frac{5}{2}}-lx^2+mx$ (at least two powers correct)	M1
	$\int x^{\frac{3}{2}}-3x+9\,dx=\frac{2}{5}x^{\frac{5}{2}}-\frac{3}{2}x^2+9x(+c)$	A1A1
		(3)
(c)	$\left[\frac{2}{5}x^{\frac{5}{2}}-\frac{3}{2}x^2+9x\right]_{\frac{5}{2}}^{4}=\left(\frac{2}{5}\times4^{\frac{5}{2}}-\frac{3}{2}\times4^2+9\times4\right)-\left(\frac{2}{5}\left(\frac{5}{2}\right)^{\frac{5}{2}}-\frac{3}{2}\left(\frac{5}{2}\right)^2+9\left(\frac{5}{2}\right)\right)=...$ $(=24.8-17.077...=7.722...)$	M1
	Area R = "12.33" – "7.722" = …	M1
	= awrt 4.6	A1
		(3)
		(10 marks)

Notes:

(a)

B1: For using h = 0.25 or equivalent. Score for sight of h = 0.25 or $\frac{1}{2}\times\frac{1}{4}$ or just $\frac{1}{8}$ outside the brackets.

M1: Correct bracket structure for the trapezium rule. Look for "h/2"(first + last y values + 2(attempt at sum of remaining values)). If one value is missing, allow the M as a slip, but M0 if 1st or last value repeated. Must be y values used. Incorrect/invisible external bracketing may be recovered by implication.

A1: Correct bracket "h/2"{…} with all 7 y values used, though allow a minor miscopy of a term if it is clear the correct value is meant. May be implied by the correct answer.

A1: awrt 12.33

(b)

M1: Attempts to integrate. Power to increase by 1 in at least two terms.

A1: Any two terms correct.

A1: Fully correct. No need for constant of integration. Need not be simplified.

(c)

M1: Applies limits of 2.5 and 4 to their answer to (b) to get the area under C_2. Limits may be either way around for this mark. May be implied by awrt 7.72 and allow if scored from a calculator.

M1: Subtracts their area under C_2 from the answer to (a). Must be subtracting a positive number from "12.33", so if the limits were the wrong way round (or a negative answer was found in error), the answer must be made positive before subtracting.
A1: for awrt 4.6 following a correct answer to (a).

Question	Scheme	Marks
6(a)	Way 1: Eqn is $(x-3)^2+(y+4)^2-9-16+k=0$	M1
	So radius must be 4 $\Rightarrow 25-k=16$	M1
	$\Rightarrow k=9$	A1
		(3)
	Way 2: $y=0 \Rightarrow x^2-6x+k=0$ has one solution	M1
	$\Rightarrow 6^2-4\times1\times k=0$	M1
	$\Rightarrow k=9$	A1
		(3)
(b)	C intersects x axis at (3,0)	B1
	Intersects y axis when $y^2+8y+9=0 \Rightarrow y=...$ or $y="-4"\pm\sqrt{"16"-"9"}$ Or uses base of triangle is $2\sqrt{"16"-"9"}$	M1
	Area $RST=\frac{1}{2}\times2\sqrt{7}\times3$	M1
	$=3\sqrt{7}$	A1
		(4)
		(7 marks)

Notes:

(a) Way 1

M1: Attempts to complete square on both terms. May be implied by a centre of $(\pm3,\pm4)$

M1: Uses a valid method to find *k*, e.g. *y* coordinate of centre and sets $"9"+"16"-k$ = |their "4"|^2. Can be implied by sight of k = (their x coordinate of centre)^2

A1: Correct value for *k*

Way 2

M1: Substitutes equation for *x* axis ($y = 0$) into the circle equation.

M1: Sets the discriminant of the resulting quadratic to 0 (since as a tangent there is only one point of intersection).

A1: Correct value for *k*

(b)

B1: Identifies or implies the coordinates where circle touches the *x* axis. Probably from having completed the square on the equation.

M1: Attempts the *y* coordinates of the intersections with the *y* axis, or the distance between these two points. E.g. may set $x = 0$ in the circle equation, or may use centre is 3 units from *y* axis etc.

M1: Uses a correct method for the area of the triangle *RST*. Longer methods are possible, such as use of cosine rule to find an angle in the triangle, followed by $\frac{1}{2}ab\sin C$, or may use the shoelace method, e.g. $\frac{1}{2}\begin{vmatrix}3 & 0 & 0\\ 0 & -4+\sqrt{7} & -4-\sqrt{7}\end{vmatrix}=\frac{1}{2}\left|3\left(-4+\sqrt{7}\right)+0+0-0-0-3\left(-4-\sqrt{7}\right)\right|$

A1: Correct answer.

Question	Scheme	Marks
7(a)	$2=\log_3 3^2$ oe seen or implied by working	B1
	$3\log_3(2x-1)=\log_3(2x-1)^3$ or $\log_3 "3^2" + \log_3(14x-25)=\log_3\left("3^2"(14x-25)\right)$	M1
	$\Rightarrow (2x-1)^3 = 9(14x-25)$	dM1
	$\Rightarrow 8x^3-12x^2+6x-1=126x-225$ $\Rightarrow 8x^3-12x^2-120x+224=0 \Rightarrow 2x^3-3x^2-30x+56=0\,*$	A1*
		(4)
(b)	$2(\pm4)^3-3(\pm4)^2-30(\pm4)+56=\ldots$	M1
	$2(-4)^3-3(-4)^2-30(-4)+56=-128-48+120+56=0$ Hence −4 is a root of the equation.	A1
		(2)
Alt (b)	$2x^3-3x^2-30x+56=(x\pm4)\left(2x^2+\ldots\pm14\right)$	M1
	$2x^3-3x^2-30x+56=(x+4)\left(2x^2-11x+14\right)$ so −4 is a root of the equation.	A1
		(2)
(c)	$2x^3-3x^2-30x+56=0 \Rightarrow (x+4)\left(2x^2+\ldots\pm14\right)=0$	M1
	$(x+4)\left(\underline{2x^2-11x+14}\right)=0$	A1
	$(x+4)(2x-7)(x-2)=0 \Rightarrow x=\ldots$	dM1
	(Equation not defined for $x=-4$ so) solutions are 2 and $\frac{7}{2}$	A1
		(4)
		(10 marks)

Notes:

(a)

B1: States or implies $2=\log_3 3^2$ or equivalent. May be gained via $\log_3 f(x)=2 \Rightarrow f(x)=9$

M1: Uses a correct log law, either power law on $3\log_3(2x-1)=\log_3(2x-1)^3$ or sum law on $\log_3 "3^2" + \log_3(14x-25)=\log_3\left("3^2"(14x-25)\right)$ or equivalent work (e.g. difference law if combining the two log terms on one side first).

dM1: Uses correct log work to remove the logs. Allow slips, but all log work must have been correct.

A1*: Expands and simplifies to the given cubic. Must see an intermediate step between the factorised sides and final answer.

(b)

M1: Substitutes ±4 into the cubic and attempts to evaluate it.

A1: Evaluates to 0 with intermediate working shown, and gives conclusion that −4 is a root.

Alt:

M1: Attempts to take a factor of $(x \pm 4)$ out of the equation or uses long division. Look for first term $2x^2$ and last term ± 14 in the quadratic factor.

A1: Correct factorisation with conclusion. If via long division, all work must be correct with zero remainder found.

(c)

M1: Attempts to take a factor of $(x+4)$ out of the cubic, or attempts long division by $x + 4$

A1: Correct quadratic factor (either by factorisation or by long division)

Note: the first two marks may be awarded for work seen in part (b).

dM1: Depends on first M. Solves the resulting quadratic (any means).

A1: Correct two solutions **only** identified. Withhold if −4 is listed as a solution.

Since the question says “hence, using algebra and showing each step of your working”, solutions derived from a calculator with no working shown score no marks. The intermediate quadratic must be attempted to gain marks in part (c)

Question	Scheme	Marks
8(i)	$3\sin(\theta+30°)=7\cos(\theta+30°)\Rightarrow \tan(\theta+30°)=\frac{7}{3}$	B1
	$\theta+30°=\arctan\left(\frac{7}{3}\right)\quad(=66.8°)$	M1
	$\theta=36.8°$ or $216.8°$ (awrt)	A1A1
		(4)
(ii)	(a) $3\sin^3 x=5\sin x-7\sin x\cos x\Rightarrow 3\sin x(1-\cos^2 x)=5\sin x-7\sin x\cos x$	M1
	$\Rightarrow \sin x(5-7\cos x+3\cos^2 x-3)=0$	
	$\Rightarrow \sin x(3\cos^2 x-7\cos x+2)=0$	A1
	(b) $\sin x=0\Rightarrow x=0$	B1
	$\Rightarrow \sin x(3\cos x-1)(\cos x-2)=0\Rightarrow \cos x=...$	M1
	$3\cos x=1\Rightarrow x=\arccos\left(\frac{1}{3}\right)=(\pm)1.23$	M1
	Both of $x=$ awrt $-1.23,\ 1.23$	A1
		(6)
		(10 marks)

Notes:

(i)

Note: answers only scores no marks.

B1: Divides through by $\cos(\theta+30°)$ to get correct equation in tan.

M1: Applies arctan to find a value for $\theta+30°$. If using other approaches it is for correct work to achieve one value for $\theta+30°$ or just θ as appropriate. (Allow if "radians" are used.)

A1: One of awrt $36.8°$ or awrt $216.8°$

A1: Both of awrt $36.8°$ and awrt $216.8°$ with no other solutions in the range.

(ii)(a)

M1: Uses $\sin^2 x=1-\cos^2 x$ in the equation and gathers terms on one side and factors out the $\sin x$ (Allow the M if the $\sin x$ is cancelled)

A1: Achieves suitable form. Accept non-zero multiples of the quadratic in $\cos x$. (Allow if $\sin x$ is cancelled and put back in later).

(b)

B1: For $x = 0$ as one solution.

M1: Solves the quadratic in $\cos x$, usual rules (may be implied by correct answers).

M1: Takes arccos of at least one value that has modulus less than 1 which is a root of their quadratic. Allow if in degrees, but must reach a value.

A1: Both solutions from cosine equation correct and no others in the range.

	Alternatives to (i)	
8(i) Alt 1	$\sin(\theta+30°)=\frac{7}{\sqrt{58}}$ or $\cos(\theta+30°)=\frac{3}{\sqrt{58}}$	**B1**
	$(3\sin(\theta+30°))^2=7(\cos(\theta+30°))^2 \Rightarrow ..\sin^2(\theta+30°)=..(1-\sin^2(\theta+30°))$ $\Rightarrow \sin(\theta+30°)=.. \Rightarrow \theta+30°=...$ Or $(3\sin(\theta+30°))^2=7(\cos(\theta+30°))^2 \Rightarrow ..(1-\cos^2(\theta+30°)=..\cos^2(\theta+30°)$ $\Rightarrow \cos(\theta+30°)=.. \Rightarrow \theta+30°=...$	**M1**
	$\theta=36.8°$ or $216.8°$ (awrt)	**A1A1**
		(4)

Alt:

B1: For reaching $\sin(\theta+30°)=\frac{7}{\sqrt{58}}$ or $\cos(\theta+30°)=\frac{3}{\sqrt{58}}$

M1: For a complete method of squaring both sides, applying $\sin^2(...)+\cos^2(...)=1$ to achieve an equation in just sin or cos and then applying the inverse function to their solution to achieve one value for $\theta+30°$ or just θ as appropriate. . (Allow if "radians" are used.)

A1A1: As above, must have rejected any extra solutions from squaring.

8(i) Alt 2	$3\sin(\theta+30°)=7\cos(\theta+30°)=\frac{3\sqrt{3}}{2}\sin\theta+\frac{3}{2}\cos\theta=\frac{7\sqrt{3}}{2}\cos\theta-\frac{7}{2}\sin\theta$ oe	**B1**
	$3\sin(\theta+30°)=7\cos(\theta+30°)\Rightarrow$ $3(\sin\theta\cos30°\pm\cos\theta\sin30°)=7(\cos\theta\sin30°\pm\sin\theta\sin30°)$ $\Rightarrow \frac{\sin\theta}{\cos\theta}=\tan\theta=...\Rightarrow \theta=\arctan(..)=...$	**M1**
	$\theta=36.8°$ or $216.8°$ (awrt)	**A1A1**
		(4)

Alt 2:
B1: Correct application of compound angle formula and trig ratios to reach
$\frac{3\sqrt{3}}{2}\sin\theta+\frac{3}{2}\cos\theta=\frac{7\sqrt{3}}{2}\cos\theta-\frac{7}{2}\sin\theta$ **oe**
M1: Attempts compound angle formula (allow sign slips), rearranges to make $\tan\theta$ the subject and applies arctan to find at least one value for θ . (Allow if "radians" are used.)
A1A1: As main scheme.

Question	Scheme	Marks
9(a)	$V = hl^2 \Rightarrow 250000 = hl^2$ or $l = \dfrac{500}{\sqrt{h}}$ oe (may use e.g. $hl = \sqrt{250\,000h}$)	B1
	$S = l^2 + 4hl = \dfrac{"250000"}{h} + 4h \times \dfrac{"500"}{\sqrt{h}}$	M1
	$S = \dfrac{250000}{h} + 2000\sqrt{h}$ *	A1*
		(3)
(b)	$\dfrac{dS}{dh} = -\dfrac{250000}{h^2} + 2000 \times \dfrac{1}{2}h^{-\frac{1}{2}}$ oe	M1A1
	$\dfrac{dS}{dh} = 0 \Rightarrow -\dfrac{250000}{h^2} + 2000 \times \dfrac{1}{2}h^{-\frac{1}{2}} = 0 \Rightarrow h^k = ...$	dM1
	$\Rightarrow h^{\frac{3}{2}} = 250 \Rightarrow h = ...$	ddM1
	$h = 250^{\frac{2}{3}}$	A1
		(5)
(c)	$\dfrac{d^2S}{dh^2} = \dfrac{500000}{h^3} - 500h^{-\frac{3}{2}}$	M1
	$\left.\dfrac{d^2S}{dh^2}\right\rvert_{h=39.7} = 6 > 0$ hence gives the minimum value.	A1
		(2)
		(10 marks)

Notes:

(a)

B1: Correct equation linking h and l seen or implied by substitution.

M1: Attempts the surface area and substitutes to eliminate l. Allow $l^2 + 2(hl + hl)$ or $2(l^2 + hl + hl)$ (or equivalents) for the attempt at surface area.

A1*: Achieves the given result with no errors.

(b)

Allow if $\dfrac{dS}{dh}$ is missing or called $\dfrac{dy}{dx}$ throughout.

M1: Attempts derivative of the area function – power decreased by one in at least one term.

A1: Correct derivative. Need not be simplified.

dM1: Sets derivative equal to zero and attempts to solve as far as a power of h equal to something. Depends on first M

ddM1: Depends on both M marks, attempts to solve an equation with a fractional index to find h

A1: $h = 250^{\frac{2}{3}}$ or states $k = \dfrac{2}{3}$

(c)

M1: Attempts the second derivative, form $\frac{A}{h^3} - Bh^{-\frac{3}{2}}$ reached (A,B > 0). Note: the question specifies by further differentiation, so first derivative test attempts score M0.

A1: Second derivative correct and substitutes *h* to get awrt 6, and makes correct conclusion referring to sign (>0 or positive). Must see the substitution as it is not obviously positive but allow if expression is not fully simplified as long as it is clear it is positive (e.g. 8 – 2 > 0).

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