Mark Scheme (Results)

October 2023

Pearson Edexcel International Advanced Level in Pure Mathematics (WMA14) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2+bx+c)=(x+p)(x+q),\text{ where }|pq|=|c|$, leading to $x = \ldots$

   $(ax^2+bx+c)=(mx+p)(nx+q),\text{ where }|pq|=|c|\text{ and }|mn|=|a|$, leading to $x = \ldots$

2. Formula

   Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

   Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$(2-5x)^{-2}=\frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$ or e.g. $\frac{8}{4\left(1-\frac{5x}{2}\right)^{2}}$	B1
	$=8\times\frac{1}{4}\underline{\underline{\left(1+(-2)\times\left(-\frac{5x}{2}\right)+\frac{(-2)\times(-3)}{2!}\times\left(-\frac{5x}{2}\right)^{2}+\frac{(-2)\times(-3)\times(-4)}{3!}\times\left(-\frac{5x}{2}\right)^{3}...\right)}}$	M1A1
	$\frac{8}{(2-5x)^{2}}=2+10x+\frac{75}{2}x^{2}+125x^{3}...$	A1
		(4)
Alt (a) by direct expansion	$8\times(2-5x)^{-2}$ $=8\times\underline{\underline{\left(2^{-2},+(-2)2^{-3}(-5x)^{1}+\frac{-2\times-3}{2!}2^{-4}(-5x)^{2}+\frac{-2\times-3\times-4}{3!}2^{-5}(-5x)^{3}\right)}}$	B1, $\underline{\underline{\text{M1A1}}}$
	$\frac{8}{(2-5x)^{2}}=2+10x+\frac{75}{2}x^{2}+125x^{3}...$	A1
(b)	$\lvert x\rvert<\frac{2}{5}$ o.e.	B1
		(1)
		(5 marks)

(a)

B1: For taking out a factor of 2^{-2} or $\frac{1}{4}$ **from** $(2-5x)^{-2}$ to obtain e.g. $\frac{1}{4}(1\pm...)^{-2}$, $2^{-2}(1\pm...)^{-2}$.

May be implied by a constant term of 2 or by e.g. $2(1\pm...)^{-2}$

The "8" is likely to be present but it is not required for this mark.

M1: For the form of the binomial expansion $(1+ax)^{-2}$

Requires the correct structure for either term three or term four. Allow a slip on the sign.

So allow for either $\frac{(-2)\times(-3)}{2}(\pm ax)^{2}$ or $\frac{(-2)\times(-3)\times(-4)}{3!}(\pm ax)^{3}$ where $a\neq 1$, could be –5 but must be the "a" in their $"2"(1\pm ax)^{-2}$.

Condone missing brackets around the "ax" for this mark.

A1: Any unsimplified or simplified but correct form of the binomial expansion for $\left(1-\frac{5x}{2}\right)^{-2}$

Ignore the factor preceding the bracket for this mark and ignore any extra terms if found.

Score for $1+(-2)\times\left(-\frac{5x}{2}\right)+\frac{(-2)\times(-3)}{2!}\times\left(-\frac{5x}{2}\right)^{2}+\frac{(-2)\times(-3)\times(-4)}{3!}\times\left(-\frac{5x}{2}\right)^{3}$ o.e.

Brackets must be present unless they are implied by later work. Allow $\left(\frac{5x}{2}\right)^{2}$ for $\left(-\frac{5x}{2}\right)^{2}$.

The simplified form is $1+5x+\frac{75}{4}x^{2}+\frac{125}{2}x^{3}+...$ Allow as a list of terms.

A1: cao $2+10x+\frac{75}{2}x^{2}+125x^{3}...$ This must be simplified and allow as a list of terms.

Allow equivalents for $\frac{75}{2}$ e.g. 37.5 and ignore any extra terms.

Do **not** isw and mark the final answer. E.g. Fully correct work leading to $2+10x+\frac{75}{2}x^2+125x^3...$

followed by $=4+20x+75x^2+250x^3...$ loses the final mark.

<u>Alternative to (a) by direct expansion</u>

B1: This is awarded for $(2-5x)^{-2}=2^{-2}+....$ which may be implied by a final answer of 2 + …

M1: For the form of the expansion of $(2-5x)^{-2}$

Requires the correct structure for either term three or term four. Allow a slip on the sign.

So allow for either $\frac{(-2)\times(-3)}{2}(2)^{-4}(\pm 5x)^2$ or $\frac{(-2)\times(-3)\times(-4)}{3!}(2)^{-5}(\pm 5x)^3$

Condone missing brackets around the $5x$.

A1: Any unsimplified or simplified but correct form of the binomial expansion for $(2-5x)^{-2}$

Ignore the factor preceding the bracket for this mark and ignore any extra terms if found.

Score for $2^{-2}+(-2)2^{-3}(-5x)^1+\frac{-2\times-3}{2!}2^{-4}(-5x)^2+\frac{-2\times-3\times-4}{3!}2^{-5}(-5x)^3$ o.e.

Brackets must be present unless they are implied by later work.

The simplified form is $\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+\frac{125}{8}x^3+...$ Allow as a list of terms.

A1: cao $2+10x+\frac{75}{2}x^2+125x^3...$ This must be simplified and allow as a list of terms.

Allow equivalents for $\frac{75}{2}$ e.g. 37.5 and ignore any extra terms.

Do **not** isw and mark the final answer. E.g. Fully correct work leading to $2+10x+\frac{75}{2}x^2+125x^3...$

followed by $=4+20x+75x^2+250x^3...$ loses the final mark.

Note: Correct or partially correct answers with <u>no working</u> in (a) should be sent to review.

(b)

B1: $|x|<\frac{2}{5}$ oe e.g. $-\frac{2}{5}<x<\frac{2}{5}$, $x>-\frac{2}{5}$ and $x<\frac{2}{5}$, $\left(-\frac{2}{5},\frac{2}{5}\right)$

But **not** $-\frac{2}{5}<|x|<\frac{2}{5}$, $x<\frac{2}{5}$, $\left|\frac{5x}{2}\right|<1$, $\left|-\frac{5x}{2}\right|<1$, $-1<\frac{5x}{2}<1$

Question Number	Scheme	Marks
2(a)	States or uses $S = 6x^2$ **or** $\frac{dS}{dt} = 4$	B1
	States or uses $S = 6x^2$ **and** $\frac{dS}{dt} = 4$	B1
	Attempts to use $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} \Rightarrow 4 = 12x \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = ...$	M1
	$\Rightarrow \frac{dx}{dt} = \frac{1}{3x}$ oe	A1
		(4)
(a) Alt	States or uses $S = 6x^2$ **or** $\frac{dS}{dt} = 4$	B1
	States or uses $S = 6x^2$ **and** $\frac{dS}{dt} = 4$	B1
	$\frac{dS}{dt} = 4 \Rightarrow S = 4t + c \Rightarrow S = 6x^2 \Rightarrow 6x^2 = 4t + c \Rightarrow 12x\frac{dx}{dt} = 4 \Rightarrow \frac{dx}{dt} = ...$ or $S = 6x^2 \Rightarrow 6x^2 = 4t + c \Rightarrow 12x = 4\frac{dt}{dx} \Rightarrow \frac{dx}{dt} = ...$	M1
	$\Rightarrow \frac{dx}{dt} = \frac{1}{3x}$ oe	A1
(b)	States or uses $\frac{dV}{dx} = 3x^2$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3x^2 \times \frac{1}{3x} = x$	M1
	$\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$	A1
		(3)
(b) Alt	$V = x^3,\ S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}} \Rightarrow \frac{dS}{dV} = 4V^{-\frac{1}{3}}$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dS} \times \frac{dS}{dt} = \frac{1}{4V^{-\frac{1}{3}}} \times 4$	M1
	$\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$	A1
		(7 marks)

(a) **Condone the use of different variables provided the intention is clear.**
This will most likely be the use of A rather than S for surface area.

B1: States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

B1: States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

M1: Attempts to use $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ or equivalent with their $\frac{dS}{dx}$ and $\frac{dS}{dt}$ where $\frac{dS}{dx} = kx$ and $\frac{dS}{dt}$ is a constant, in an attempt to find $\frac{dx}{dt}$. May be implied by their working.

A1: $\frac{dx}{dt} = \frac{1}{3x}$ oe e.g. $\frac{1/3}{x}, \frac{4}{12x}$ from correct work and apply isw once a correct answer is seen.

Allow correct work leading to e.g. $k = \frac{1}{3}$.

The "$\frac{dx}{dt} =$" must appear at some point in their working for this mark.

(a) **Alt.**

B1: States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

B1: States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$ (ignore any units associated with either)

Note that $S = 6x^2$ may be implied by $\frac{dS}{dx} = 12x$

M1: Integrates $\frac{dS}{dt} = 4$ to obtain $S = 4t + c$ or $S = 4t$ and replaces S in terms of x and differentiates wrt t or wrt x to obtain $\alpha x \frac{dx}{dt} = \beta$ or $\alpha x = \beta \frac{dt}{dx}$

A1: $\frac{dx}{dt} = \frac{1}{3x}$ oe e.g. $\frac{1/3}{x}, \frac{4}{12x}$ from correct work and apply isw once a correct answer is seen.

Note that $\frac{dx}{dt} = \frac{1}{3x}$ sometimes comes from incorrect work so check their method.

This can be scored from obtaining $S = 4t + c$ or $S = 4t$ earlier.

The "$\frac{dx}{dt} =$" must appear at some point in their working for this mark.

(b)

B1: States or uses $\frac{dV}{dx} = 3x^2$ Allow this mark to score anywhere in the question.

M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ or equivalent with their $\frac{dV}{dx}$ and $\frac{dx}{dt}$ which may be implied.

A1: $\frac{dV}{dt} = V^{\frac{1}{3}}$ which may be implied by e.g. $\frac{dV}{dt} = x,\ x = V^p \Rightarrow p = \frac{1}{3}$.

The "$\frac{dV}{dt} =$" must appear at some point in their working for this mark.

(b) **Alt:**

B1: States or uses $\frac{\mathrm{d}S}{\mathrm{d}V} = 4V^{-\frac{1}{3}}$ (from $S = 6x^2$, $V = x^3 \Rightarrow S = 6V^{\frac{2}{3}}$)

M1: Attempts to use $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}S} \times \frac{\mathrm{d}S}{\mathrm{d}t}$ or equivalent with their $\frac{\mathrm{d}V}{\mathrm{d}S}$ and $\frac{\mathrm{d}S}{\mathrm{d}t}$ which may be implied.

A1: $\frac{\mathrm{d}V}{\mathrm{d}t} = V^{\frac{1}{3}}$ which may be implied by e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = x,\ x = V^p \Rightarrow p = \frac{1}{3}$

The "$\frac{\mathrm{d}V}{\mathrm{d}t} =$" must appear at some point in their working for this mark.

Note that a common incorrect response involves using $S = x^2$ rather than $S = 6x^2$, giving:

(a) $S = x^2,\ \frac{\mathrm{d}S}{\mathrm{d}t} = 4,\ \frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow 4 = 2x \times \frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{x}$ scoring B1B0M1A0

(b) $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2,\ \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = 3x^2 \times \frac{2}{x} = 6x = 6V^{\frac{1}{3}}$ scoring B1M1A0

Question Number	Scheme	Marks
3(i)	$\int x^2 e^{2x}\, dx = \frac{1}{2}x^2 e^{2x} - \int x\, e^{2x}\, dx$	M1
	$= \frac{1}{2}x^2 e^{2x} - \left\{\frac{1}{2}x e^{2x} - \int \frac{e^{2x}}{2}\, dx\right\}$	M1
	$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{e^{2x}}{4}$	A1
	$\int_0^4 x^2 e^{2x}\, dx = \left[\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{e^{2x}}{4}\right]_0^4 = 8e^8 - 2e^8 + \frac{e^8}{4} - \frac{1}{4}$	M1
	$= \frac{25e^8}{4} - \frac{1}{4}$	A1
		(5)

(i) Condone the omission of "dx" throughout

M1: Attempts to integrate by parts once to achieve $Px^2 e^{2x} - Q\int x\, e^{2x} dx, \quad P, Q > 0$

M1: Attempts to integrate xe^{2x} to achieve $\lambda x e^{2x} \pm \mu \int e^{2x}\, dx, \ \lambda > 0$

A1: $\int x^2 e^{2x}\, dx = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{e^{2x}}{4}$ which may be unsimplified and isw once a correct answer is seen.

Watch for $\int x^2 e^{2x}\, dx = \frac{1}{2}x^2 e^{2x} - \left(\frac{1}{2}x e^{2x} - \frac{e^{2x}}{4}\right)(+c)$ which scores all 3 marks. ISW after sight of this.

Watch for D & I method which may be seen. You will just see the answer here.

D	**I**
x^2	$+ \quad e^{2x}$
$2x$	$- \quad \frac{1}{2}e^{2x}$
2	$+ \quad \frac{1}{4}e^{2x}$
0	$\frac{1}{8}e^{2x}$

$$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x}$$

Score M2 for $Ax^2 e^{2x} \pm Bxe^{2x} \pm Ce^{2x}, \ A > 0$

and then A1 for $\frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{e^{2x}}{4}(+c)$

M1: Attempts to substitute **both** limits into an expression of the form $Ax^2 e^{2x} \pm Bxe^{2x} \pm Ce^{2x}, \ A > 0,$ subtracts either way round, and simplifies to the form $\alpha e^8 + \beta, \ \alpha, \beta \neq 0$ where α and β are numeric. (Allow if the e^8 terms are not combined)

A1: $\frac{25e^8}{4} - \frac{1}{4}$ or exact simplified equivalent e.g. $\frac{1}{4}\left(25e^8 - 1\right), \ \frac{25}{4}\left(e^8 - \frac{1}{25}\right)$. Do **not** condone "+ c" here.

(ii)	**1.** $u = 2x-1 \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$ **OR** **2.** $u = (2x-1)^2 \Rightarrow \frac{du}{dx} = 4(2x-1)$ or e.g. $du = 4(2x-1)dx$ **OR** **3.** $u = 2x \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$	B1
	1. $u = 2x-1 \Rightarrow \int \frac{4x}{(2x-1)^2}\,dx = \int \frac{2u+2}{u^2} \times \frac{1}{2}\,du$ **OR** **2.** $u = (2x-1)^2 \Rightarrow \int \frac{4x}{(2x-1)^2}\,dx = \int \frac{2\sqrt{u}+2}{u} \times \frac{1}{4\sqrt{u}}\,du$ **OR** **3.** $u = 2x \Rightarrow \int \frac{4x}{(2x-1)^2}\,dx = \int \frac{2u}{(u-1)^2} \times \frac{1}{2}\,du$	M1 A1
	1. $\int \left(\frac{1}{u} + \frac{1}{u^2}\right) du = \ln u - \frac{1}{u}$ **OR** **2.** $\frac{1}{2}\int \left(\frac{1}{u} + \frac{1}{u^{\frac{3}{2}}}\right) du = \frac{1}{2}\ln u - \frac{1}{\sqrt{u}}$ **OR 3.** $\int \frac{u}{(u-1)^2}\,du = -u(u-1)^{-1} + \int \frac{1}{u-1}\,du = -u(u-1)^{-1} + \ln(u-1)$	**d**M1 A1
	1. Uses limits $u = 5$ to $u = 20$: $\left(\ln 20 - \frac{1}{20}\right) - \left(\ln 5 - \frac{1}{5}\right)$ **OR** **2.** Uses limits $u = 25$ to $u = 400$: $\left(\frac{1}{2}\ln 400 - \frac{1}{20}\right) - \left(\frac{1}{2}\ln 25 - \frac{1}{5}\right)$ **OR** **3.** Uses limits $u = 6$ to $u = 21$: $\left(\ln 20 - \frac{21}{20}\right) - \left(\ln 5 - \frac{6}{5}\right)$	M1
	$= \frac{3}{20} + \ln 4$	A1
		(7)
		(12 marks)

Note that 'u' = $2x - 1$ is by far the most common approach.

(ii)

B1: Uses an appropriate substitution and differentiates **correctly**. See scheme for 3 examples.

Note that for case **2.** $u=\left(2x-1\right)^2 \Rightarrow x=\dfrac{\sqrt{u}+1}{2} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{1}{4}u^{-\frac{1}{2}}$ is also correct and scores B1

M1: Valid attempt to change the integral to one in "u" to obtain an integrand of the correct form:
This requires e.g.

$$\textbf{1. } k\int \frac{au+b}{u^2}\,\mathrm{d}u,\ a,b\neq 0 \quad \textbf{OR} \quad \textbf{2. } k\int \frac{a\sqrt{u}+b}{u}\frac{1}{\sqrt{u}}\,\mathrm{d}u,\quad a,b\neq 0 \quad \textbf{OR} \quad \textbf{3. } k\int \frac{u}{\left(u-1\right)^2}\,\mathrm{d}u$$

Condone the omission of du as long as the form of the integrand is correct.

Note that the u's may appear "combined" e.g. as $u^{\frac{3}{2}}$ in the denominator of case **2.**

A1: A correct integrand in any form in terms of u only.
Condone the omission of du as long as the integrand is correct.

dM1: Correct form of the integration for their substitution: This requires e.g.

$$\textbf{1. } k\int \frac{au+b}{u^2}\,\mathrm{d}u \to p\ln u + qu^{-1} \qquad \textbf{OR} \qquad \textbf{2. } k\int \frac{a\sqrt{u}+b}{u}\frac{1}{\sqrt{u}}\,\mathrm{d}u \to p\ln u + \frac{q}{\sqrt{u}}$$

$$\textbf{OR} \quad \textbf{3. } k\int \frac{u}{\left(u-1\right)^2}\,\mathrm{d}u \to \alpha u\left(u-1\right)^{-1} + \beta\ln\left(u-1\right)$$

Depends on the previous method mark.

A1: Fully correct integration. Ignore any limits for this mark.

Note that there may be other acceptable attempts to integrate e.g. in case **1.**:
$\int \frac{u+1}{u^2}\,\mathrm{d}u = \int \left(u+1\right)u^{-2}\,\mathrm{d}u = -\left(u+1\right)\frac{1}{u} + \int \frac{1}{u}\,\mathrm{d}u = -\left(u+1\right)\frac{1}{u} + \ln u$
In such cases, award M1 for a fully correct method and A1 if correct.

M1: Attempts to use the correct limits for their substitution.
Need to see evidence that both correct changed limits have been substituted and the resulting expressions subtracted either way round.
There must have been some attempt to integrate however poor having used substitution.
The substitution must involve a transformation so e.g. $u = x$ is not acceptable.

Alternatively, reverses the substitution and uses the x limits 3 and $\dfrac{21}{2}$.

A1: $\ln 4+\dfrac{3}{20}$ or e.g. $\ln 4+0.15$ and apply isw if necessary.

<u>Special Case: Candidates who answer part (b) without any form of substitution can score a special case of B0M0A0 (no substitution) then M1A1dM1A1</u>

e.g. Partial fractions:

$$\frac{4x}{(2x-1)^2} \equiv \frac{A}{2x-1}+\frac{B}{(2x-1)^2} \equiv \frac{2}{2x-1}+\frac{2}{(2x-1)^2}$$

$$\int \frac{2}{2x-1}+\frac{2}{(2x-1)^2}\,\mathrm{d}x = \ln(2x-1)-\frac{1}{(2x-1)}$$

$$\left[\ln(2x-1)-\frac{1}{2x-1}\right]_3^{\frac{21}{2}} = \ln 20-\frac{1}{20}-\ln 5+\frac{1}{5}=\frac{3}{20}+\ln 4$$

Mark as B0M0A0 then:

M1 for $\int \frac{A}{2x-1}+\frac{B}{(2x-1)^2}\,\mathrm{d}x = \alpha\ln(2x-1)+\beta\frac{1}{(2x-1)}$

A1 for $\int \frac{2}{2x-1}+\frac{2}{(2x-1)^2}\,\mathrm{d}x = \ln(2x-1)-\frac{1}{(2x-1)}(+c)$

dM1 Substitutes both correct limits and subtracts either way round. **Depends on first M.**

A1: $\ln 4+\frac{3}{20}$ or e.g. $\ln 4+0.15$ and apply isw if necessary.

or Parts:

$$\int \frac{4x}{(2x-1)^2}\,\mathrm{d}x = -2x(2x-1)^{-1}+\ln(2x-1)$$

$$=\left[-2x(2x-1)^{-1}+\ln(2x-1)\right]_3^{\frac{21}{2}} = -\frac{21}{20}+\ln 20+\frac{6}{5}-\ln 5=\frac{3}{20}+\ln 20$$

M1 for $\int \frac{4x}{(2x-1)^2}\,\mathrm{d}x = \alpha x(2x-1)^{-1}+\beta\ln(2x-1)$

A1 for $\int \frac{4x}{(2x-1)^2}\,\mathrm{d}x = -2x(2x-1)^{-1}+\ln(2x-1)(+c)$

dM1 Substitutes both correct limits and subtracts either way round. **Depends on first M.**

A1: $\ln 4+\frac{3}{20}$ or e.g. $\ln 4+0.15$ and apply isw if necessary.

If you find any attempts using a combination of methods that include substitution e.g.

$$\frac{4x}{(2x-1)^2} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} \equiv \frac{2}{2x-1} + \frac{2}{(2x-1)^2}$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2} \, \mathrm{d}x = \left[\ln(2x-1)\right] + \int \frac{2}{u^2} \frac{\mathrm{d}u}{2} \qquad (u = 2x-1)$$

$$\int \frac{2}{2x-1} + \frac{2}{(2x-1)^2} \, \mathrm{d}x = \left[\ln(2x-1)\right]_3^{\frac{21}{2}} + \left[-\frac{1}{u}\right]_5^{20} = \ln 20 - \ln 5 - \frac{1}{20} + \frac{1}{5} = \frac{3}{20} + \ln 4$$

OR

$$\int \frac{4x}{(2x-1)^2} \, \mathrm{d}x = \left[-2x(2x-1)^{-1}\right] + \int \frac{2}{2x-1} \, \mathrm{d}x$$

$$= \left[-2x(2x-1)^{-1}\right] + \int \frac{2}{u} \frac{\mathrm{d}u}{2} \qquad (u = 2x-1)$$

$$= \left[-2x(2x-1)^{-1}\right]_3^{\frac{21}{2}} + \left[\ln u\right]_5^{20} = -\frac{21}{20} + \frac{6}{5} + \ln 20 - \ln 5 = \frac{3}{20} + \ln 20$$

if fully correct as above then score full marks otherwise send to review.

Question Number	Scheme	Marks
4(a)	Assume that there exists a positive number k such that $k+\frac{9}{k}<6$	B1
	$k+\frac{9}{k}<6 \Rightarrow k^2+9<6k \Rightarrow k^2-6k+9<0$ **or** $k+\frac{9}{k}<6 \Rightarrow k+\frac{9}{k}-6<0 \Rightarrow \left(\sqrt{k}...\right)\left(\sqrt{k}...\right)<0$ **or** $k+\frac{9}{k}<6 \Rightarrow \left(k+\frac{9}{k}\right)^2<36 \Rightarrow k^2+18+\frac{81}{k^2}-36<0$	M1
	$\Rightarrow (k-3)^2<0$ **or** $\Rightarrow \left(\sqrt{k}-\frac{3}{\sqrt{k}}\right)^2<0$ **or** $\Rightarrow \left(k-\frac{9}{k}\right)^2<0$	A1
	But numbers squared are $\geqslant 0$, hence $k+\frac{9}{k}\geqslant 6$	A1*
		(4)
(b)	E.g. When $k=-3,\ -3+\frac{9}{-3}\left(=-6\right)$ which is not $\geqslant 6$	B1
		(1)
(b) Alt	$k<0 \Rightarrow \frac{9}{k}<0 \Rightarrow k+\frac{9}{k}<0$ which is not $\geqslant 6$	B1
		(5 marks)

(a)

B1: For setting up the correct contradiction. It must include a word/words such as "assume" or "let" and must be a strict inequality. As a minimum accept just "assume/let $k+\frac{9}{k}<6$".

Condone e.g. Assume that for all positive numbers k, $k+\frac{9}{k}<6$

M1: Starting from $k+\frac{9}{k}<6$ or $k+\frac{9}{k}\leqslant 6$ or $k+\frac{9}{k}=6$, either

- multiplies by k and attempts to reach $k^2...<0$ or $k^2...\leqslant 0$ or $k^2...=0$
- collects terms to one side to reach $k+\frac{9}{k}\pm 6 \leqslant / < / = 0$ and attempts to factorise to $\left(\sqrt{k}...\right)\left(\sqrt{k}...\right)\leqslant / < / = 0$
- squares both sides and collects terms to reach $k^2...<0$ or $k^2...\leqslant 0$ or $k^2...=0$

A1: Reaches $(k-3)^2...0$ or e.g. $\left(\sqrt{k}-\frac{3}{\sqrt{k}}\right)^2...0$ or e.g. $\left(k-\frac{9}{k}\right)^2...0$ where … is $<$ or $\leqslant$ or $=$

A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason: "numbers squared are $\geqslant 0$ "this is impossible", "numbers squared are not negative"
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

<u>Alternatives for A1A1*:</u>

1. Using discriminant:

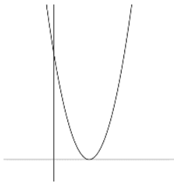
A1: $b^2-4ac=6^2-4\times 9=0$ so one root and 'positive' quadratic

A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason e.g. so $k^2-6k+9\geqslant 0$ or curve is on or above the *x*/*k* axis
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

2. Via sketch:

A1: Sketches $y=k^2-6k+9$:



A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason e.g. graph always on or above (or never below) the *x*-axis so $k^2-6k+9\geqslant 0$
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

3. Via differentiation:

A1: $\frac{\mathrm{d}\left(k^2-6k+9\right)}{\mathrm{d}k}=2k-6=0\Rightarrow k=3\Rightarrow k^2-6k+9=0$ so minimum at (3, 0)

A1*: For a fully correct proof.
Requires

- correct calculations/algebra
- a reason e.g. so $k^2-6k+9\geqslant 0$ or curve is on or above the *x*/*k* axis
- a minimal conclusion. E.g. hence $k+\frac{9}{k}\geqslant 6$, hence proven, QED, etc. but not just "contradiction"

If candidates use another variable for *k*, e.g. *x* or *m* then withhold the final mark unless they revert to *k* in their conclusion.

This is also acceptable (working backwards):

Assume $k+\frac{9}{k}<6$ B1

$(k-3)^2 \geqslant 0 \Rightarrow k^2-6k+9 \geqslant 0 \Rightarrow k-6+\frac{9}{k} \geqslant 0$ M1

Condone > for $\geqslant$

From $(k-3)^2 \geqslant 0$ expands and divides by k

$\Rightarrow k+\frac{9}{k} \geqslant 6$ A1

Condone > for $\geqslant$

This contradicts the assumption and so $k+\frac{9}{k} \geqslant 6$ A1*

Note: Attempts that use the contradiction: "Assume there are negative numbers for which $k+\frac{9}{k} \geqslant 6$" generally score no marks but use review if necessary.

(b)

B1: Chooses a negative number, shows the result of substituting into $k+\frac{9}{k}$ and gives a

minimal conclusion e.g. It is not necessary to evaluate e.g. $-3+\frac{9}{-3}$ is sufficient.

- which is not $\geqslant 6$
- is < 6

(b)**Alt:**

B1: States that if k < 0 then $k+\frac{9}{k}<0$ and gives a minimal conclusion e.g.

- which is not $\geqslant 6$
- is < 6

There must be no incorrect work and no contradictory/incorrect statements.

Do not allow arguments that refer to *k* = 0 being not valid.

Question Number	Scheme	Marks
5(a)	$y^3 - x^2 + 4x^2 y = k$	
	$y^3 \to 3y^2 \dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$4x^2 y \to 8xy + 4x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$y^3 - x^2 + 4x^2 y = k \Rightarrow 3y^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} - 2x + 8xy + 4x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	A1
	$\Rightarrow \left(3y^2 + 4x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x - 8xy$	M1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x - 8xy}{3y^2 + 4x^2}$	A1
		(5)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -1$ at P	M1
	Uses $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \pm 1$ and $y = x$ to set up and solve equation in x, y or 'p' $\Rightarrow -1 = \dfrac{2p - 8p^2}{3p^2 + 4p^2} \Rightarrow p = 2$	M1 A1
	e.g. $k = "2"^3 - "2"^2 + 4 \times "2"^3$	**dd**M1
	$k = 36$	A1
		(5)
		(10 marks)

(a) Allow equivalent notation for the $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ e.g. y'

M1: Differentiates $y^3 \to Py^2 \dfrac{\mathrm{d}y}{\mathrm{d}x}$

M1: Uses the product rule to differentiate $4x^2 y$ and obtains $Qxy + Rx^2 \dfrac{\mathrm{d}y}{\mathrm{d}x}$

A1: Correct differentiation including the " = 0" which may be implied by subsequent work.

Note that some candidates have a spurious $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ at the start (as their intention to differentiate) and this can be ignored for the first 3 marks so condone $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3y^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} - 2x + 8xy + 4x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$

Allow versions such as $3y^2\mathrm{d}y - 2x\mathrm{d}x + 8xy\mathrm{d}x + 4x^2\mathrm{d}y = 0$

M1: Dependent upon having achieved two <u>different</u> terms in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, one from each of the terms y^3 and $4x^2 y$.

Look for $(\ldots \pm \ldots)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ldots \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ which may be implied by their working.

For those candidates who had a spurious $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x - 8xy}{3y^2 + 4x^2}$ or exact equivalent e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{8xy - 2x}{-3y^2 - 4x^2}$

(b)

M1: States or uses $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$ which may be implied by their working e.g.

$$\frac{2x-8xy}{3y^2+4x^2} = -1,\ \frac{8xy-2x}{3y^2+4x^2} = 1,\ \frac{3y^2+4x^2}{8xy-2x} = 1 \text{ etc.}$$

Do not award for e.g. "Gradient = –1"

M1: Uses $y = x$ or $x = y$ or $y = x =$ "p" in their $\frac{\mathrm{d}y}{\mathrm{d}x}$ sets $= \pm 1$ and attempts to solve to obtain a value for their variable.

e.g. $\frac{2x-8x^2}{3x^2+4x^2} = \pm 1 \Rightarrow x = ...,\ \frac{2y-8y^2}{3y^2+4y^2} = \pm 1 \Rightarrow y = ...,\ \frac{2p-8p^2}{3p^2+4p^2} = \pm 1 \Rightarrow p = ...,$

A1: For x, y or "p" = 2 at P

ddM1: Uses their non-zero value for x, y or "p" to obtain a value for k using the given equation of the curve.

Depends on both previous method marks.

A1: $k = 36$

Correct answer only with no working scores no marks.

Question Number	Scheme	Marks
6(a)	$\left(2, 3, -7\right)$	B1
		(1)
(b)	Attempts $\begin{pmatrix}1\\2\\2\end{pmatrix}\bullet\begin{pmatrix}4\\-1\\8\end{pmatrix}=1\times4+2\times-1+2\times8=\left(18\right)$	M1
	Attempts $\mathbf{a}.\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$: $18=\sqrt{1^2+2^2+2^2}\times\sqrt{4^2+\left(-1\right)^2+8^2}\cos\theta$	**d**M1
	$\cos\theta=\dfrac{2}{3}$	A1
		(3)
(b) Alt	e.g. $\pm\left(\begin{pmatrix}4\\-1\\8\end{pmatrix}-\begin{pmatrix}1\\2\\2\end{pmatrix}\right)=\pm\begin{pmatrix}3\\-3\\6\end{pmatrix}$	M1
	$3^2+3^2+6^2=1^2+2^2+2^2+4^2+\left(-1\right)^2+8^2-2\sqrt{1^2+2^2+2^2}\times\sqrt{4^2+\left(-1\right)^2+8^2}\cos\theta$	dM1
	$\cos\theta=\dfrac{2}{3}$	A1
(c)	Uses $\lambda = 6$ to find length PQ E.g. $\overrightarrow{PQ}=6\mathbf{i}+12\mathbf{j}+12\mathbf{k}\Rightarrow PQ=\sqrt{6^2+12^2+12^2}=\left(18\right)$ Or $PQ=6\times\sqrt{1^2+2^2+2^2}=\left(18\right)$	M1
	Area $QPR=\dfrac{1}{2}ab\sin C=\dfrac{1}{2}\times18^2\times\sqrt{1-\left(\dfrac{2}{3}\right)^2}=54\sqrt{5}$	M1 A1
		(3)
(d)	Attempts a correct method of finding at least one value for μ e.g. $\left(4\mu\right)^2+\mu^2+\left(8\mu\right)^2=6^2+12^2+12^2$ $\Rightarrow\mu=\dfrac{"18"}{\sqrt{4^2+\left(-1\right)^2+8^2}}=\left(\pm\right)2$	M1
	Attempts one correct position $\mathbf{r}=\begin{pmatrix}2\\3\\-7\end{pmatrix}\pm2\begin{pmatrix}4\\-1\\8\end{pmatrix}$	**d**M1
	Possible coordinates $\left(10,1,9\right)$ and $\left(-6,5,-23\right)$	A1
		(3)
		(10 marks)

(a)

B1: Correct coordinates or position vector e.g. as shown or $x = 2$, $y = 3$, $z = -7$ or $2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ or $\begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix}$

Condone $(2\mathbf{i}, 3\mathbf{j}, -7\mathbf{k})$

(b)

M1: Attempts $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ condoning slips. Note that any non-zero multiples of these vectors can be used.

If the method is not explicit then this mark may be implied by 2 correct components.

dM1: Full attempt at $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where **a** and **b** are the direction vectors or multiples of them.

Depends on the first method mark.

Note that $\cos\theta = \dfrac{4-2+16}{\sqrt{1^2+2^2+2^2}\sqrt{4^2+1^2+8^2}}$ would imply both method marks.

A1: $\cos\theta = \dfrac{2}{3}$

(b) Alternative using the cosine rule:

M1: Attempts $\pm\left(\alpha\begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} - \beta\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\right)$

dM1: Full attempt at the cosine rule using the appropriate lengths.

Depends on the first method mark.

A1: $\cos\theta = \dfrac{2}{3}$

(c)

M1: Uses $\lambda = 6$ to find the length of PQ. E.g. uses Pythagoras to calculate $|6\mathbf{i}+12\mathbf{j}+12\mathbf{k}|$ or $6|\mathbf{i}+2\mathbf{j}+2\mathbf{k}|$

If the method is not explicit then the attempt at $\overrightarrow{PQ}$ may be implied by 2 correct components.

M1: Full attempt at $\dfrac{1}{2}ab\sin C$ where $a = b = |\overrightarrow{PQ}|$ and C is their θ which may be attempted in decimals.

Their θ or $\sin\theta$ must follow an attempt to use their $\cos\theta$.

A1: $54\sqrt{5}$ or exact equivalent e.g. $\dfrac{162\sqrt{5}}{3}$

(d)

M1: Attempts a **correct** method of finding at least one value for μ.

Pythagoras must be used correctly and both sides of their equation must be consistent e.g.

$(4\mu)^2 - \mu^2 + (8\mu)^2 = 18^2$ and $(4\mu)^2 + \mu^2 + (8\mu)^2 = 18$ both score M0

Must be correct work here so must be using the length of *PQ* not *OQ*.

dM1: Attempts one correct position $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} \pm 2 \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ **Depends on the first method mark.**

A1: Gives both possible coordinates $(10,\ 1,\ 9)$ and $(-6,\ 5,\ -23)$

Allow as coordinates or vectors or as $x = \ldots, y = \ldots, z = \ldots$

Question Number	Scheme	Marks
7(a)	Substitutes $x = 3$ and $t = 0$ into $x=\dfrac{k(9t+5)}{4t+3}$	M1
	$\Rightarrow 3=\dfrac{5k}{3}\Rightarrow k=1.8$ *	A1*
		(2)
(b)	4050	B1
		(1)
(c)	$\dfrac{3}{x(9-2x)}\equiv\dfrac{A}{x}+\dfrac{B}{9-2x}$	M1
	Either $A=\dfrac{1}{3}$ or $B=\dfrac{2}{3}$	A1
	$\dfrac{3}{9x-2x^2}\equiv\dfrac{1}{3x}+\dfrac{2}{3(9-2x)}$	A1
		(3)
(d)	$3\dfrac{dx}{dt}=x(9-2x)\Rightarrow\displaystyle\int\dfrac{3}{x(9-2x)}dx=\int dt$ **Or** $3\dfrac{dx}{dt}=x(9-2x)\Rightarrow\displaystyle\int\dfrac{1}{x(9-2x)}dx=\int\dfrac{1}{3}dt$ $\Rightarrow\dfrac{1}{3}\ln x-\dfrac{1}{3}\ln(9-2x)=t(+c)$ **or** $\dfrac{1}{9}\ln x-\dfrac{1}{9}\ln(9-2x)=\dfrac{1}{3}t(+c)$	M1 A1ft
	Substitutes $t=0, x=3\Rightarrow c=0$	M1
	$\dfrac{1}{3}\ln x-\dfrac{1}{3}\ln(9-2x)=t\Rightarrow\left(\dfrac{x}{9-2x}\right)=e^{3t}$	**dd**M1
	$\left(\dfrac{9-2x}{x}\right)=e^{-3t}\Rightarrow\dfrac{9}{x}-2=e^{-3t}\Rightarrow x=\dfrac{9}{2+e^{-3t}}$ *	A1*
		(5)
(e)	4500	B1
		(1)
		(12 marks)

(a)

M1: Substitutes $x = 3$ and $t = 0$ into the given equation

A1*: Shows that k =1.8 (oe e.g. $\dfrac{9}{5}$) with no errors and with at least one correct line of the form $ak=b$

e.g. $3=\dfrac{k(9\times0+5)}{4\times0+3}\Rightarrow k=1.8,\ \ 3=\dfrac{k(0+5)}{0+3}\Rightarrow k=1.8,\ \ 3=\dfrac{5k}{3}\Rightarrow k=1.8$ **all score M1A1**

Alternative by verification:

M1: Substitutes k = 1.8 and t = 0 into the given equation

A1*: Shows that x = 3 with at least one correct intermediate line and a concluding statement that this is 3000

Special cases:

1. $x=\dfrac{k(9\times 0+5)}{4\times 0+3}\Rightarrow k=1.8$ scores M1A0

2. Substitutes x = 3000 and t = 0 into the given equation to obtain k = 1800 (hence k = 1.8) scores M1A0

(b)

B1: 4050 cao. Allow 4.05 thousand.

(c)

M1: Sets $\dfrac{3}{x(9-2x)}\equiv\dfrac{A}{x}+\dfrac{B}{9-2x}$ or equivalent e.g. $3\equiv A(9-2x)+Bx$

A1: One correct value for 'A' or 'B' or one correct fraction.

A1: Correct **<u>fractions</u>** e.g. $\dfrac{3}{x(9-2x)}\equiv\dfrac{1}{3x}+\dfrac{2}{3(9-2x)}$ oe e.g. $\dfrac{1/3}{x}+\dfrac{2/3}{(9-2x)}$, $\dfrac{1}{3x}+\dfrac{2}{27-6x}$

This mark is for the correct partial fractions not for the values of the constants.

Award once the correct fractions are seen and allow if seen in (d) if not seen here.

(d)

M1: Attempts to separate the variables and integrate to obtain a kt term and one of $\alpha\ln\beta x$ or $\gamma\ln\delta(9-2x)$.

Condone missing brackets e.g. $\ln 9-2x$

The ln terms must come from a partial fraction of the form $\dfrac{A}{x}$ or $\dfrac{B}{9-2x}$.

A1ft: $\dfrac{1}{3}\ln x-\dfrac{1}{3}\ln(9-2x)=t(+c)$ but ft on their A and B so award for $A\ln x-\dfrac{B}{2}\ln(9-2x)=t(+c)$ oe

There is no requirement for + c

Brackets must be present unless they are implied by later work.

Note that there are various correct alternatives here e.g. $\dfrac{1}{3}\ln 3x-\dfrac{1}{3}\ln(27-6x)=t(+c)$

M1: Sets t = 0 and x = 3 in an attempt to find "c" having made some attempt to integrate at least one of their partial fractions to obtain a ln term.

If this step is not attempted only the first two marks are available in this part.

They can "state" e.g. c = 0 or e.g. K = 1 provided it follows correct work and there was a "+ c".

ddM1: **Dependent upon both previous Method marks.**

It is for using correct work to remove the ln's having found a constant of integration.

A1*: Correct work to reach the given answer showing all necessary steps.

The scheme shows one such way with acceptable minimal working.

Note that some candidates may rearrange first before finding the constant of integration e.g.

$$3\frac{\mathrm{d}x}{\mathrm{d}t}=x\left(9-2x\right)\Rightarrow\int\frac{3}{x\left(9-2x\right)}\mathrm{d}x=\int\mathrm{d}t$$

$$\Rightarrow\frac{1}{3}\ln x-\frac{1}{3}\ln\left(9-2x\right)=t+c$$

$$\Rightarrow\frac{1}{3}\ln\frac{x}{9-2x}=t+c\Rightarrow\ln\frac{x}{9-2x}=3t+d\Rightarrow\frac{x}{9-2x}=K\mathrm{e}^{3t}$$

$$t=0 \text{ and } x=3\ \Rightarrow K=1$$

$$\frac{x}{9-2x}=\mathrm{e}^{3t}\ \Rightarrow 9\mathrm{e}^{3t}-2x\mathrm{e}^{3t}=x\Rightarrow x\left(2\mathrm{e}^{3t}+1\right)=9\mathrm{e}^{3t}$$

$$\Rightarrow x=\frac{9\mathrm{e}^{3t}}{\left(2\mathrm{e}^{3t}+1\right)}=\frac{9}{2+\mathrm{e}^{-3t}}\ *$$

In these cases, the first 2 marks are as already defined and then award M1 at the point t = 0 and x = 3 are used in an attempt to find "c" and then **dd**M1 for using correct work to remove the ln's and A1* for correct work to reach the given answer showing all necessary steps.

As in the main scheme, if there is no "+ c" then only the first 2 marks are available in (d).

Note that in (d) it is possible to start again e.g.

$$3\frac{\mathrm{d}x}{\mathrm{d}t}=x\left(9-2x\right)\Rightarrow\int\frac{1}{x\left(9-2x\right)}\mathrm{d}x=\int\frac{1}{3}\mathrm{d}t$$

$$\frac{1}{x(9-2x)}\equiv\frac{A}{x}+\frac{B}{9-2x}\equiv\frac{1}{9x}+\frac{2}{9(9-2x)}$$

etc.

(e)

B1: 4500 cao or 4.5 thousand

Question Number	Scheme	Marks
8(a)	3π	B1
		(1)
(b)	$\dfrac{dy}{dx}=\dfrac{-2\sin t}{6-6\cos 2t}$	M1 A1
	$=\dfrac{-2\sin t}{6-6\left(1-2\sin^2 t\right)}=\dfrac{-2\sin t}{12\sin^2 t}=-\dfrac{1}{6}\operatorname{cosec} t$	M1 A1*
		(4)
(c)	$P=\left(\dfrac{3\pi}{2}-3,\sqrt{2}\right)$	B1
	E.g. $y-\sqrt{2}=-\dfrac{\sqrt{2}}{6}\left(0-\left(\dfrac{3\pi}{2}-3\right)\right)\Rightarrow y=...$ or $\sqrt{2}=-\dfrac{\sqrt{2}}{6}\left(\dfrac{3\pi}{2}-3\right)+c\Rightarrow c=...$	M1
	$\dfrac{\pi\sqrt{2}}{4}+\dfrac{\sqrt{2}}{2}$ o.e	A1
		(3)
(d)(i)	Attempts $y^2\dfrac{dx}{dt}=4\cos^2 t\left(6-6\cos 2t\right)$	M1 A1
	$=\left(2\cos 2t+2\right)\left(6-6\cos 2t\right)=6\left(2-2\cos^2 2t\right)=6\left(2-\left(1+\cos 4t\right)\right)$	**d**M1
	Volume $=\displaystyle\int_0^{\frac{\pi}{2}}\pi y^2\frac{dx}{dt}dt=\int_0^{\frac{\pi}{2}}6\pi\left(1-\cos 4t\right)dt$	A1
(ii)	$\left[6\pi t-\dfrac{6\pi\sin 4t}{4}\right]_0^{\frac{\pi}{2}}$, $=3\pi^2$	M1, A1
		(6)
		(14 marks)

(a)

B1: For 3π. Condone $(3\pi, 0)$ and allow $x = 3\pi$. Remember to check the diagram as the 3π may appear there.

Do not allow decimals here.

(b)

M1: Attempts to differentiate $x(t)$ and $y(t)$ and calculates $\frac{dy}{dx}$ by using $\frac{dy}{dt}\div\frac{dx}{dt}$.

Condone poor differentiation but one of the terms must be of the correct form so it requires $x\to\alpha+\beta\cos 2t$ or $y\to A\sin t$

A1: $\dfrac{dy}{dx}=\dfrac{-2\sin t}{6-6\cos 2t}$ or equivalent e.g. $\dfrac{dy}{dx}=\dfrac{-2\sin t}{6-6\left(\cos^2 t-\sin^2 t\right)}$

M1: Uses appropriate trigonometry to reach $\frac{dy}{dx} = \alpha \text{ cosec } t$.

Condone sign errors only with any identities that are used.

A1: $\frac{dy}{dx} = -\frac{1}{6}\text{cosec } t$

You can condone poor notation along the way e.g. $\frac{dy}{dx} = \frac{-2\sin t}{6-6\left(\cos^2 - \sin^2\right)}$ as long as the intention is clear.

An alternative for the final M1A1: $\frac{-2\sin t}{6-6\cos 2t} = \lambda \text{cosec } t \Rightarrow \lambda = \frac{-2\sin^2 t}{6-6\cos 2t} = \frac{-2\sin^2 t}{12\sin^2 t} = -\frac{1}{6}$

(c)

B1: At P, $x = \frac{3\pi}{2} - 3$, $y = \sqrt{2}$ or exact equivalents

May be implied by being seen embedded in their tangent equation.

M1: Full method of finding the y coordinate of N

Requires:

- substitution of $t = \frac{\pi}{4}$ into their $\frac{dy}{dx} = \alpha \text{ cosec } t$ to find gradient of tangent
- use of their $\left(\frac{3\pi}{2} - 3, \sqrt{2}\right)$ (which may be inexact) in the formation of the equation of the tangent or the substitution of these values into $y = mx + c$ with the coordinates correctly placed.
- the setting of $x = 0$ as well as finding the y value or rearranging to find "c"

A1: $\frac{\pi\sqrt{2}}{4} + \frac{\sqrt{2}}{2}$ or exact simplified equivalent. E.g. $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$.

Does not need to be identified as a value for y so just look for the correct exact expression (may be $c = \ldots$).

(d) (i) Note that the first 3 marks only involve the consideration of $y^2 \frac{dx}{dt}$ i.e. not an integral.

Condone poor notation e.g. $\cos t^2$ for $\cos^2 t$ as long as it is recovered and the intention is clear.

M1: Attempts $y^2 \frac{dx}{dt}$ with their $\frac{dx}{dt}$ condoning slips

A1: $y^2 \frac{dx}{dt} = 4\cos^2 t\left(6-6\cos 2t\right)$ oe. Allow e.g. $\left(2\cos t\right)^2\left(6-6\cos 2t\right)$ with both sets of brackets present unless they are implied by subsequent work.

dM1: A full method to form an expression in terms of $\cos 4t$.

Condone sign errors only with any identities that are used. **Depends on the first method mark.**

There are many ways of achieving the result so you will need to check their work carefully.

E.g.: $$4\cos^2 t\left(6-6\cos 2t\right) = 4\cos^2 t\left(12\sin^2 t\right) = 12\sin^2 2t = 6-6\cos 4t$$

or

$$4\cos^2 t\left(6-6\cos 2t\right) = 4\cos^2 t\left(12\sin^2 t\right) = 48\cos^2 t - 48\cos^4 t$$

$$\cos 4t = \cos^2 2t - \sin^2 2t = \left(2\cos^2 t - 1\right)^2 - 4\sin^2 t\cos^2 t = 8\cos^4 t - 8\cos^2 t + 1$$

$$\Rightarrow 48\cos^2 t - 48\cos^4 t = 6\left(1-\cos 4t\right)$$

A1: Volume = $\int_0^{\frac{\pi}{2}} 6\pi\left(1-\cos 4t\right)\mathrm{d}t$. All correct with correct limits and including the "d*t*".

(d)(ii)

M1: For $\int \beta\left(1-\cos 4t\right)\mathrm{d}t \to \left[\beta\left(t-\frac{\sin 4t}{4}\right)\right]$. Allow with their β, a made up β or the letter β.

A1: $3\pi^2$ following **full marks in (d)(i)** and **correct integration** and **correct limits.**

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