Mark Scheme (Results)

Summer 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers may score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).
- **Ignoring subsequent work**

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=\dots$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\dots$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\dots$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".
General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this, e.g. in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed – i.e. giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the rule may allow the mark to be awarded before the final answer is given.

June 2306
4PM1 Paper 1
Mark Scheme

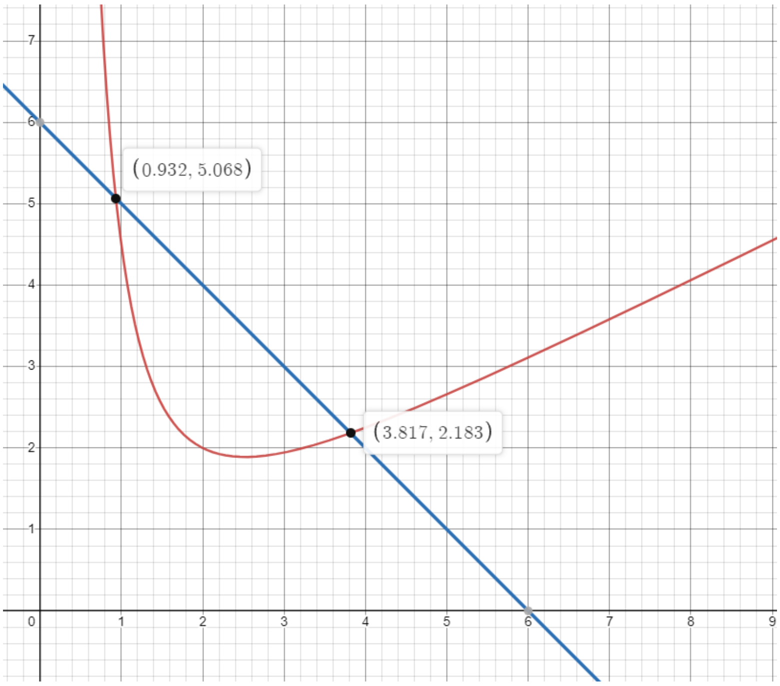
Question	Scheme	Marks
1(a)	$d = 3$ and $a = 5$	B1
	$\sum_{r=1}^{n}(3r+2) = \frac{n}{2}(2\times5+(n-1)3) = \frac{n}{2}(3n+7)*$	M1A1 cso [3]
	ALT	
	$\sum_{r=1}^{n}(3r+2) = 3\sum_{r=1}^{n} r + 2\sum_{r=1}^{n} 1$	[B1
	$\sum_{r=1}^{n}(3r+2) = \frac{3}{2}n(n+1)+2n = \frac{3n^2+3n}{2}+\frac{4n}{2} = \frac{3n^2+7n}{2} = \frac{n}{2}(3n+7)$	M1A1]
(b)	$\sum_{r=10}^{40}(3r+2) = \frac{40}{2}(3\times40+7) - \frac{9}{2}(3\times9+7) = [2540-153] = 2387$	M1A1 [2]
		Total 5 marks

Note: Part (b) can be answered simply by entering the expression in the question into a permissible calculator. A solution seen with no working is M0.

Part	**Mark**	**Notes**
(a)	**B1**	For the correct value of *a* and *d*
	M1	For using the correct summation formula with their values of *a* and *d*
	A1	For achieving $\sum_{r=1}^{n}(3r+2)=\frac{n}{2}(3n+7)$ with no errors
	ALT 1 – Uses first plus last formula	
	B1	For the correct value of $a = 5$ and the final value $l = 3n + 2$
	M1	For the correct use of the first plus last summation formula $\sum_{r=1}^{n}(3r+2)=\frac{n}{2}(5+[3n+2])=\frac{n}{2}(3n+7)$
	A1	For achieving $\sum_{r=1}^{n}(3r+2)=\frac{n}{2}(3n+7)$ with no errors
	ALT 2 – Uses standard results	
	B1	For stating $\sum_{r=1}^{n}(3r+2)=3\sum_{r=1}^{n}r+2\sum_{r=1}^{n}1$
	M1	For using the standard expressions $\sum_{r=1}^{n}(3r+2)=\frac{3}{2}n(n+1)+2n$
	A1	For simplifying to the required form. $\sum_{r=1}^{n}(3r+2)=\frac{3n^2+3n}{2}+\frac{4n}{2}=\frac{3n^2+7n}{2}=\frac{n}{2}(3n+7)$ *
(b)	**M1**	Uses the given formula with n = 40 and with n = 9 (allow n = 10 for this mark) [Note: use of n = 10 will give a value of 2355] Allow alternative correct methods
	A1	For the correct sum of 2387
	ALT 1 – Uses first + last summation formula	
	M1	$U_{10}=32,\quad U_{40}=122,\quad n=31\ \ (\text{allow 30 for this mark})$ $\sum_{r=10}^{40}(3r+2)=\frac{31}{2}(32+122)=...$ [Note: Use of n = 30 will give a value of 2310]
	A1	For the correct sum of 2387
	ALT 2 – Finds new first term and uses the summation formula	
	M1	$U_{10}=32\quad n=31\ \ (\text{allow 30 for this mark})$ $\sum_{r=10}^{40}(3r+2)=\frac{31}{2}(2\times 32+(31-1)3)=...$ [Note: Use of n = 30 will give a value of 2265]
	A1	For the correct sum of 2387

Question	**Scheme**	**Marks**
2	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin 2x \times \frac{1}{2} \times 2 \times (3+2x)^{-\frac{1}{2}} + (3+2x)^{\frac{1}{2}} \times 2 \times \cos 2x$	M1A1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin 2x}{\sqrt{3+2x}} + 2 \times \sqrt{(3+2x)} \cos 2x = \frac{\sin 2x + 2(3+2x)\cos 2x}{\sqrt{3+2x}} = \ldots$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin 2x + (6+4x)\cos 2x}{\sqrt{3+2x}}$ *	A1 cso
	Where *A* = 6 and *B* = 4	[5]
		Total 5 marks

Mark	Notes
M1	For an attempt to apply product rule: An attempt is as follows. • The formula used must be correct. $\frac{\mathrm{d}y}{\mathrm{d}x} = uv' + vu'$ • Minimally acceptable differentiation used within a correct formula. $\sin 2x \to k\cos 2x$ and $\sqrt{3+2x} \to l(3+2x)^{-\frac{1}{2}}$ where *k* and *l* are a positive integers and $k, l \neq 0$ (You will see *l* = 1 without other working which is correct)
A1	One term correct – (simplification not required) and the other minimally acceptable. Either $\sin 2x \times \frac{1}{2} \times 2 \times (3+2x)^{-\frac{1}{2}}$ Or $(3+2x)^{\frac{1}{2}} \times 2 \times \cos 2x$
A1	Fully correct – simplification not required $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin 2x \times \frac{1}{2} \times 2 \times (3+2x)^{-\frac{1}{2}} + (3+2x)^{\frac{1}{2}} \times 2 \times \cos 2x$
dM1	For **correct** use of $\sqrt{3+2x}$ or $(3+2x)^{\frac{1}{2}}$ as a common denominator. **NB:** This mark is dependent on the first M mark scored.
A1 **cso**	For the correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ as shown with no erroneous or missing working. Allow embedded values. E.g., $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin 2x + (6+4x)\cos 2x}{\sqrt{3+2x}}$ is dM1A1

Question	Scheme	Marks
3	$\frac{x}{2}+\frac{4}{x^2}=Ax+B \Rightarrow x^3(1-2A)-2Bx^2+8=0$	M1
	$x^3(1-2A)-2Bx^2+8=3x^3-12x^2+8 \Rightarrow A=-1,\ B=6 \Rightarrow y=6-x$	M1A1
	Draws line $y=6-x$	M1
	Roots are $x=0.9,\ 3.8$	A1
		[5]
		
		Total 5 marks

Mark	Notes
M1	Removes the denominators by multiplying through by $2x^2$ This can be carried out separately by multiplying by 2 and then x^2 or vice versa. Allow one slip in their working. $\frac{x}{2}+\frac{4}{x^2}=Ax+B \Rightarrow x^3+8=2Ax^3+2Bx$
M1	Collects like terms and equates components. $x^3(1-2A)-2Bx^2+8=3x^3-12x^2+8$ and finds the equation of the straight line.
A1	The straight line is $y=6-x$
ALT 1	
M1	Divides through by $2x^2$ This can be carried out separately by dividing by 2 and then x^2 or vice versa.

	Allow one slip in their working. $3x^3-12x^2+8=0 \quad \left(\div 2x^2\right) \Rightarrow \frac{3x}{2}-6+\frac{4}{x^2}=0$
M1	Adds $(6-x)$ to both sides. $\Rightarrow \frac{x}{2}+\frac{4}{x^2}=6-x$
A1	The straight line is $y=6-x$
ALT 2	
M1 **M1**	Multiplies through $\frac{x}{2}+\frac{4}{x^2}=y \Rightarrow x^3+8=2x^2y$ **and** subtracts this from $3x^3-12x^2+8=0$ as follows to obtain $2x^3-12x^2=-2x^2y$ $3x^3-12x^2+8=0$ $x^3 \quad\quad +8=2x^2y$ $\overline{2x^3-12x^2 \quad\quad =-2x^2y}$
M1	Divides $2x^3-12x^2=-2x^2y$ by $2x^2$ to obtain a linear equation $x-6=-y$
A1	The straight line is $y=6-x$
SC: No or incorrect working to find the equation of the line. • If fully correct answers are seen without correct or any working at all and without the correct **equation** of the line seen [even if a line is drawn], award M0M0A0M0A0 • If the rounded answers to 1 dp are seen **with** the correct equation $y=6-x$, and the line drawn, award M0M0A0M1A1 • If rounded answers are seen to a greater degree of accuracy **with** the correct equation $y=6-x$ seen, and the line drawn, award M0M0A0M1A0	
M1	For drawing their straight line correctly onto the grid, provided it is of the form $y=k-x$
A1	For a value of 0.9 [allow 1.0] and 3.8 (do not award without the evidence of the line) Allow seen as coordinates (0.9, [5.1]) (3.8, [2.2]) ignore y values. Do not accept awrt in this question. Rounding must be to one decimal place.

Question	Scheme	Marks
4(a)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 4t - 16 \quad [t = 5] \quad a = 4\times 5 - 16 = 4$	M1A1 [2]
(b)	$v = 0 \Rightarrow 2t^2 - 16t + 30 = 0 \Rightarrow (t-5)(2t-6) = 0$	M1
	$\Rightarrow t_1 = 3, \ \ t_2 = 5$	A1A1
	Displacement = $\int_3^5 \left(2t^2 - 16t + 30\right) \mathrm{d}t = \left[\frac{2t^3}{3} - \frac{16t^2}{2} + 30t\right]_3^5$	M1A1
	$= \left[\frac{2\times 5^3}{3} - 8\times 25 + 30\times 5\right] - \left[\frac{2\times 3^3}{3} - 8\times 9 + 30\times 3\right] = \frac{100}{3} - 36 = -\frac{8}{3}$	M1A1
	Distance = $\frac{8}{3}$ (m)	A1ft
		[8]
	Total 10 marks	

Part	Mark	Notes
(a)	**M1**	For an attempt to differentiate the given expression and substitute *t* = 5 See General Guidance for the definition of an attempt. Do not accept an expression with any terms integrated.
	A1	Obtains the value 4
(b)	**M1**	Sets *v* = 0 and attempts to solve the equation using a correct method. Please check their work carefully. See General Guidance for acceptable ways to solve a quadratic. If they use a calculator and only *t* = 3, 5 are seen with no working – award M1A1A1
	A1	For either $t_1 = 3$ or $t_2 = 5$
	A1	For both $t_1 = 3$ and $t_2 = 5$
		No working seen to find the distance.
		SC: If a final answer of $\frac{8}{3}$ is seen with no evidence of algebraic integration, **award the final A mark only.**
	M1	For an attempt to integrate the given expression for *v* with or without limits. Ignore poor or absent notation. [ie. No integral sign] See General Guidance for the definition of an attempt. Note: Do not accept an integrated expression with any **algebraic** terms differentiated.
	A1	For the correct integrated expression unsimplified or simplified. isw incorrect simplification following a correctly integrated expression. Accept for this mark the inclusion of + *c* Ignore limits for this mark.
	M1	Substitutes their values of $t_1 = 3$ or $t_2 = 5$ into their integrated expression and attempts to evaluate. Allow the subtraction either way around. Their integral must be a changed expression. Accept even if they have differentiated it. If the final answer is not $\pm\frac{8}{3}$ and there is no explicit substitution seen, award M0.
	A1	For the correct value of $-\frac{8}{3}$ You must see the negative value for this mark. Accept awrt – 2.67 for this mark.
	A1ft	For the correct distance of $\frac{8}{3}$ (m) or exact equivalent. Do not accept 2.67 (m) but accept $2.\dot{6}$ **Note:** This is a ft mark. We will only award this mark if their value for the displacement is **negative** and the final distance is positive.

Question	Scheme	Marks
5(a)	$V = x\times 4x\times h \Rightarrow 4x^2h = 75 \Rightarrow h = \frac{75}{4x^2}$ $S = 2\left(4x^2+xh+4xh\right) = \left[8x^2+2xh+8xh\right] \Rightarrow S = 8x^2+10xh$ $\Rightarrow S = 8x^2+10x\times\frac{75}{4x^2} \Rightarrow S = 8x^2+\frac{375}{2x}$ *	B1 M1 M1A1 cso [4]
(b)(i)	$\frac{\mathrm{d}S}{\mathrm{d}x} = 16x - \frac{375}{2x^2} = 0$ $\Rightarrow 16x = \frac{375}{2x^2} \Rightarrow x^3 = \frac{375}{32} \Rightarrow x = 2.2714... \approx 2.27\ (\text{cm})$	M1 M1A1
(ii)	$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 16 + 375x^{-3} \approx 48$ Conclusion: (As *x* is always positive) $\frac{\mathrm{d}^2S}{\mathrm{d}x^2} > 0 \Rightarrow$ minimum	M1 A1ft [5]
(c)	$S = 8\times'2.27'^2 + \frac{375}{2\times'2.27'} = 123.822... \approx 124\ (\text{cm}^2)$	M1A1 [2]
		Total 11 marks

Part	Mark	Notes
(a)	**B1**	For finding the correct expression for *h* in terms of *x* Accept this mark seen anywhere in part (a) Accept also for example, $xh = \frac{75}{4x}$ or any equivalent that can be substituted into an expression for *S*.
	M1	For finding an expression for *S* is terms of *x* and *h*. Accept as a minimum $Ax^2 + Bxh$ where *A* and *B* are constants. Accept other letters for *S* at this stage. E.g *SA* etc. or even no letter at all.
	M1	For substituting their *h* into their *S*
	A1	For the correct expression for *S* as shown in the question. You must see $S = 8x^2 + \frac{375}{2x}$ including *S* although *S* seen at the top of a column of working is also fine.
(b)	**M1**	For an attempt to differentiate the given *S* which as a minimum must be of the form

		$\frac{\mathrm{d}S}{\mathrm{d}x}=16x-\frac{k}{x^2}$ where *k* is a constant.
	M1	For setting their $\frac{\mathrm{d}S}{\mathrm{d}x}=0$ and attempting to solve the equation and reaching a value of *x*. It must be a clearly differentiated expression even if they do not score the first M mark in (b). That is, they must reduce the power of at least one term by 1.
	A1	For awrt $x = 2.27$
	M1	For an attempt to differentiate their $\frac{\mathrm{d}S}{\mathrm{d}x}$ to find $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$ which must be as a minimum $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=16+lx^{-3}$ where *l* is a constant **Other methods** Please send to Review any examples of candidates who test the gradient on either side of the minimum, or who draw a sketch.
	A1ft	Concludes that as $x > 0$ so $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}>0$ and therefore *S* is a minimum when $x = 2.27$ Dependent on correct method seen. Follow through their value of *x* provided it is positive. **NB:** They do not need to evaluate $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$ to score this mark. [See above] However, if they use the correct [2.27] or an incorrect positive value of *x*, provided substitution is seen award the mark even if the final evaluation is incorrect. If no substitution is seen and either the value of *x* is incorrect or they do not obtain approximately 48, withhold this mark.
(c)	**M1**	Substitutes their $x = 2.27$ into the given *S* Follow through their value of *x* provided it is a positive value. Use of a negative value of *x* is M0. If the final value of *S* is incorrect following an incorrect value of *x* award this mark only for explicit substitution seen.
	A1	For awrt $S = 124$ (cm^2)

Question	Scheme	Marks
6	$\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \log_2 x^3 + \dfrac{\log_2 x^2}{\log_2 4} - \dfrac{3\log_2 2}{\log_2 x} = 0$ $= 3\log_2 x + \dfrac{2\log_2 x}{\log_2 4} - \dfrac{\log_2 2^3}{\log_2 x} = 0$ $3\log_2 x + \dfrac{2\log_2 x}{2} - \dfrac{3}{\log_2 x} = 0$ $\Rightarrow 3(\log_2 x)^2 + (\log_2 x^2)^2 - 3 = 0$ $\Rightarrow 4(\log_2 x)^2 = 3 \Rightarrow (\log_2 x)^2 = \dfrac{3}{4} \Rightarrow \log_2 x = \pm\sqrt{\dfrac{3}{4}}$ $\Rightarrow x = 2^{\sqrt{\frac{3}{4}}} \approx 1.82$ or $x = 2^{-\sqrt{\frac{3}{4}}} \approx 0.549$	M1 M1 B1 M1 M1 M1A1A1 [8]
		Total 8 marks

Mark	Notes **NB: Candidates will frequently use a substitution for their chosen log.**
Working in log base 2 **Correct answer/s with no working scores no marks.**	
M1	For changing the base of the log correctly in at least one term $\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \log_2 x^3 + \dfrac{\log_2 x^2}{\log_2 4} - \dfrac{3\log_2 2}{\log_2 x} = 0$
M1	For using the power law in at least one term. $\log_2 x^3 = 3\log_2 x$ or $\log_2 x^2 = 2\log_2 x$ or $3\log_2 2 = \log_2 2^3$ $\Rightarrow 3\log_2 x + \log_2 x - \dfrac{\log_2 2^3}{\log_2 x} = 0$
B1	For either $\log_2 4 = 2$ or $\log_2 8 = 3$
M1	For multiplying through by $\log_2 x$ $3(\log_2 x)^2 + (\log_2 x)^2 - 3 = 0$
M1	For simplifying and obtaining two values for $\log_2 x$ using a valid method. **NB:** If they discard one value at any stage do not award this mark. $4(\log_2 x)^2 - 3 = 0 \Rightarrow \log_2 x = \pm\sqrt{\dfrac{3}{4}}$
M1	For removing the log to find at least one value for x $x = 2^{\sqrt{\frac{3}{4}}}$ or $x = 2^{-\sqrt{\frac{3}{4}}}$
A1	For either awrt $x = 1.82$ or 0.549
A1	For awrt both $x = 1.82$ and 0.549

Working in log base 4	
M1	For changing the base of the log correctly in at least one term $\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \frac{\log_4 x^3}{\log_4 2} + \log_4 x^2 - \frac{3\log_4 2}{\log_4 2} = 0$
M1	For using the power law in at least one term $\frac{3\log_4 x}{\log_4 2} + 2\log_4 x - \frac{3\log_4 2}{\log_4 2} = 0$
B1	For either $\log_4 2 = \frac{1}{2}$ or $\log_4 8 = \frac{3}{2}$
M1	For multiplying through by $\log_4 x$ $6\left(\log_4 x\right)^2 + 2\left(\log_4 x^2\right)^2 - \frac{3}{2} = 0$
M1	For simplifying and obtaining two values for $\log_4 x$ $8\left(\log_4 x\right)^2 = \frac{3}{2} \Rightarrow \left(\log_4 x\right)^2 = \frac{3}{16} \Rightarrow \log_4 x = \pm\sqrt{\frac{3}{16}}$ **NB:** If they discard one value at any stage do not award this mark.
M1	For removing the log to find at least one value for *x* using a valid method. $x = 4^{\sqrt{\frac{3}{16}}} \approx ....$ or $x = 4^{-\sqrt{\frac{3}{16}}} \approx ...$
A1	For either awrt $x = 1.82$ or 0.549
A1	For awrt both $x = 1.82$ and 0.549
Working in log base *x*	
M1	For changing the base of the log correctly in at least one term $\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = \frac{\log_x x^3}{\log_x 2} + \frac{\log_x x^2}{\log_x 4} - 3\log_x 2 = 0$
M1	For using the power law in at least one term $\frac{3\log_x x}{\log_x 2} + \frac{2\log_x x}{2\log_x 2} - 3\log_x 2 = 0$
B1	For $\log_x x = 1$ $\frac{3}{\log_x 2} + \frac{1}{\log_x 2} - 3\log_x 2 = 0 \Rightarrow \frac{4}{\log_x 2} - 3\log_x 2 = 0$
M1	For multiplying through by $\log_x 2$ $4 - 3\left(\log_x 2\right)^2 = 0$
M1	For obtaining two values of $\log_x 2$ using a valid method $\left(\log_x 2 = \pm\sqrt{\frac{4}{3}}\right)$ **NB:** If they discard one value at any stage do not award this mark.
M1	For removing the log to find at least one value for *x*. $2 = x^{\sqrt{\frac{4}{3}}} \Rightarrow x = 2^{\sqrt{\frac{3}{4}}} \approx ....$ or $2 = x^{-\sqrt{\frac{4}{3}}} \Rightarrow x = 2^{-\sqrt{\frac{3}{4}}} \approx ....$
A1	For either awrt $x = 1.82$ or 0.549

A1	For awrt both $x = 1.82$ and 0.549

Question	**Scheme**	**Marks**
7(a)	$y=\sqrt{\frac{e^{4x}}{2x-3}}=e^{2x}\left(2x-3\right)^{-\frac{1}{2}}$ $y=\frac{e^{2x}}{\sqrt{2x-3}}\Rightarrow\frac{dy}{dx}=\frac{2\left(2x-3\right)^{\frac{1}{2}}e^{2x}-2\times\frac{1}{2}\times e^{2x}\left(2x-3\right)^{-\frac{1}{2}}}{\left[\left(2x-3\right)^{\frac{1}{2}}\right]^{2}}$ $\frac{dy}{dx}=\frac{\frac{2\left(2x-3\right)e^{2x}-2\times\frac{1}{2}\times e^{2x}}{\left(2x-3\right)^{\frac{1}{2}}}}{2x-3}=\frac{2\left(2x-3\right)e^{2x}-2\times\frac{1}{2}\times e^{2x}}{\left(2x-3\right)^{\frac{3}{2}}}$ $=\frac{e^{2x}\left(4x-7\right)}{\left(2x-3\right)^{\frac{3}{2}}}$ $\Rightarrow\delta y\approx\frac{e^{2x}\left(4x-7\right)}{\left(2x-3\right)^{\frac{3}{2}}}\delta x*$	B1 M1A1A1 M1 A1 A1cso [7]
(b)	When $x=2.5,\ \ \delta x=\frac{0.2}{100}\times2.5=0.005$ $\delta y\approx\frac{e^{2\times2.5}\left(4\times2.5-7\right)}{\left(2\times2.5-3\right)^{\frac{3}{2}}}\times0.005$ $\Rightarrow\delta y\approx0.79$	B1 M1 A1 [3]
		Total 10 marks

Part	**Mark**	**Notes**
(a)	**If any candidate attempts part (a) using Chain Rule – please send to Review.**	
	B1	Simplifies the equation into a form which can be differentiated. For example, Award this mark for correct subsequent use in differentiation. $y=\sqrt{\frac{e^{4x}}{2x-3}}=e^{2x}\left(2x-3\right)^{-\frac{1}{2}}$ or $\frac{e^{2x}}{\left(2x-3\right)^{\frac{1}{2}}}$ or even $\frac{\left(e^{4x}\right)^{\frac{1}{2}}}{\left(2x-3\right)^{\frac{1}{2}}}$ or $\frac{\left(e^{4x}\right)^{\frac{1}{2}}}{\sqrt{\left(2x-3\right)}}$
	Uses Quotient rule. – **NB – This is a 'show' question. Check every line of working.**	
	M1	• The denominator must be correct and squared. • There must be an attempt to differentiate both terms • The two terms in the numerator must be subtracted either way around. Minimally acceptable differentiation is as follows: $e^{2x}\to2e^{2x},\quad\left(2x-3\right)^{\frac{1}{2}}\to k\left(2x-3\right)^{-\frac{1}{2}}$ Allow $\left(e^{4x}\right)^{\frac{1}{2}}\to4e^{4x}\times\frac{1}{2}\times\left(e^{4x}\right)^{-\frac{1}{2}}$

	A1	One term must be fully correct Either $2(2x-3)^{\frac{1}{2}}e^{2x}$ or $-2\times\frac{1}{2}\times e^{2x}(2x-3)^{-\frac{1}{2}}$
	A1	$\frac{dy}{dx}$ fully correct. Ignore poor notation and erroneous subsequent simplification. $'\frac{dy}{dx}'=\frac{2(2x-3)^{\frac{1}{2}}e^{2x}-2\times\frac{1}{2}\times e^{2x}(2x-3)^{-\frac{1}{2}}}{2x-3}$ **OR** $'\frac{dy}{dx}'=\frac{(2x-3)^{\frac{1}{2}}\times4e^{4x}\times\frac{1}{2}\times\left(e^{4x}\right)^{-\frac{1}{2}}-2\times\frac{1}{2}\times\left(e^{4x}\right)^{\frac{1}{2}}(2x-3)^{-\frac{1}{2}}}{2x-3}$
	Uses Product Rule **NB – This is a ‘show’ question. Check every line of working.**	
	M1	The correct formula must be used. • $\frac{dy}{dx}=uv'+vu'$ • There must be an attempt to differentiate both terms $e^{2x}\to2e^{2x},\quad(2x-3)^{-\frac{1}{2}}\to k(2x-3)^{-\frac{3}{2}}$ Allow $\left(e^{4x}\right)^{\frac{1}{2}}\to4e^{4x}\times\frac{1}{2}\times\left(e^{4x}\right)^{-\frac{1}{2}}$
	A1	One term must be fully correct Either $(2x-3)^{-\frac{1}{2}}\times2e^{2x}$ or $\left(-\frac{1}{2}\times2\times(2x-3)^{-\frac{3}{2}}\right)e^{2x}$
	A1	$\frac{dy}{dx}$ fully correct. Ignore poor notation and erroneous subsequent simplification. $'\frac{dy}{dx}'=(2x-3)^{-\frac{1}{2}}\times2e^{2x}+\left(-\frac{1}{2}\times2\times(2x-3)^{-\frac{3}{2}}\right)e^{2x}$ **OR** $'\frac{dy}{dx}'=(2x-3)^{-\frac{1}{2}}\times4e^{4x}\times\frac{1}{2}\times\left(e^{4x}\right)^{-\frac{1}{2}}+\left(-\frac{1}{2}\times2\times(2x-3)^{-\frac{3}{2}}\right)\left(e^{4x}\right)^{\frac{1}{2}}$
	Simplification – Check their work carefully here.	
	M1	Quotient Rule A correct attempt to simplify the numerator by forming a fraction over $(2x-3)^{\frac{1}{2}}$ Product Rule A correct attempt to simplify by forming a fraction over $(2x-3)^{\frac{3}{2}}$
	A1	For $'\frac{dy}{dx}'=\frac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}$
	A1 **cso**	For the expression exactly as given $\delta y\approx\frac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}\delta x$
(b)	**B1**	For finding the change in x
	M1	For using the given expression to substitute the values and evaluate the expression.

		Do not accept a substitution of 0.2 or 0.2% for δx
	A1	For $'\delta y' \approx 0.79$ accept awrt 0.79 Do not penalise poor notation here. Allow dy = 0.79

	ALT 2 Uses Chain Rule	
	B1	This mark is scored when they divide through later by e^{2x} and $(2x-3)^{\frac{1}{2}}$
	M1	The correct form must be used. $y=u^{\frac{1}{2}}$ $u'=\frac{4(2x-3)\mathrm{e}^{4x}-2\mathrm{e}^{4x}}{(2x-3)^2}=\left[\frac{\mathrm{e}^{4x}(8x-14)}{(2x-3)^2}\right]$ $y'=\frac{1}{2}\left(\frac{\mathrm{e}^{4x}}{2x-3}\right)^{-\frac{1}{2}}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(\frac{\mathrm{e}^{4x}}{2x-3}\right)^{-\frac{1}{2}}\times\left(\frac{4(2x-3)\mathrm{e}^{4x}-2\mathrm{e}^{4x}}{(2x-3)^2}\right)$ Both terms must be differentiated correctly $\mathrm{e}^{4x}\to 4\mathrm{e}^{4x}$, $(2x-3)\to k$
	A2	For the correct derivative in unsimplified form. We must see this in full to determine if they simplify further correctly as it is a show question. Please award **both** A marks for the correct derivative seen.
	B1	This mark is awarded at this point. Simplifies $\left(\frac{\mathrm{e}^{4x}}{2x-3}\right)^{-\frac{1}{2}}$ to e^{2x} and $(2x-3)^{\frac{1}{2}}$ accept inverted or not.
	M1	Simplifies e^{2x} and $(2x-3)^{\frac{1}{2}}$ to obtain a denominator of $(2x-3)^{\frac{3}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(\frac{(2x-3)^{\frac{1}{2}}}{\mathrm{e}^{2x}}\right)\times\left(\frac{4(2x-3)\mathrm{e}^{4x}-2\mathrm{e}^{4x}}{(2x-3)^2}\right)=\frac{1}{2}\left(\frac{4(2x-3)\mathrm{e}^{2x}-2\mathrm{e}^{2x}}{(2x-3)^{\frac{3}{2}}}\right)$
	A1	For the correct expression with no errors.

Question	Scheme	Marks
8	$\int\left(18x^2-2x+13\right)\ \mathrm{d}x=\frac{18x^3}{3}-\frac{2x^2}{2}+13x\left(+c\right)$	M1A1
	$\mathrm{f}\left(\frac{1}{2}\right)=6\times\left(\frac{1}{2}\right)^3-\left(\frac{1}{2}\right)^2+13\times\left(\frac{1}{2}\right)+c=0\Rightarrow c=-7$	M1A1
	$\mathrm{f}\left(x\right)=6x^3-x^2+13x-7$ oe	A1
	$2x-1\overset{3x^2+x+7}{\overline{\left)6x^3-x^2+13x-7\right.}}$	M1A1
	$b^2-4ac=1^2-4\times3\times7=-83$	M1
	Conclusion: $b^2-4ac<0$ so there is only one root/intersection with the x-axis $\left[\text{at }\left(\frac{1}{2},0\right)\right]$	A1cso [9]
		Total 9 marks

Mark	Notes
M1	For integrating the given $\mathrm{f}'\left(x\right)$. + c is not required for this mark This can be simplified or unsimplified. Accept $\frac{18x^3}{3}-\frac{2x^2}{2}+13x\left(+c\right)$ or $6x^3-x^2+13x\left(+c\right)$ At least two terms must be integrated correctly, with no differentiation.
A1	For the correct integrated expression. Ignore the absence of + c
M1	For substituting $x=\frac{1}{2}$ into their integrated expression (which must include +c, otherwise it is M0) and setting equal to 0. **ALT** Some candidates are completing this step using polynomial division. If they do not have + c at this stage then it is M0. If it is clear they are attempting to find the value of c by finding a numerical value for c [even if it is incorrect], award both this mark and the next M mark provided they obtain a quotient of $3x^2+x+k$

A1	For the correct value of $c = -7$
A1	For the fully correct $\text{f}\left(x\right)=6x^3-x^2+13x-7$ all on one line. You do not need to see f(x) = … Accept even when you see this as a complete dividend in the 'bus stop' or when equating coefficients.
M1	Polynomial division For dividing their f (x) by $(2x-1)$ to achieve as a minimum $3x^2+x$. Award this for correct attempts even without either + c or a value for c. Allow also: $x-\frac{1}{2}\overset{6x^2+2x\ \left(+l\right)}{\overline{\big)6x^3-x^2+13x-7}}$ [the correct quotient is $6x^2+2x+14$] **ALT** Equating coefficients. They must achieve a correct value for A and B for the award of this mark. $6x^3-x^2+13x-7=\left(2x-1\right)\left(Ax^2+Bx+C\right)\Rightarrow A=3,\quad B=1$ OR $6x^3-x^2+13x-7=\left(x-\frac{1}{2}\right)\left(Ax^2+Bx+C\right)\Rightarrow A=6,\quad B=2$
A1	For the correct quotient/quadratic factor $3x^2+x+7$ or $6x^2+2x+14$
M1	For using b^2-4ac on their f (x) which must be a 3TQ Accept this as part of the quadratic formula. Sight of the complex roots $\left(\frac{-1\pm\left(\sqrt{83}\right)\text{i}}{6}\right)$ is M1 Allow this mark even if they do not have +c or find the correct cubic expression.
A1 **cso**	For concluding that as $b^2-4ac<0$ then there is only one root/intersection with the x-axis. Do not accept statements such as 'not possible' or 'will not factorise' without reference to a negative discriminant. Accept evidence of a statement such as $1^2<4\times3\times7\Rightarrow b^2<4ac$ oe, or embedded in a formula without explicit evaluation. Complex roots must be followed by a comment that as the roots are not real, there are no intersections with the x-axis. Quoting the complex roots without a correct explanation is A0. If they have not found +c at the start, this mark is not available as this solution is cso.

Question	Scheme	Marks
9(a)	$\cos 2A=\cos^2 A-\sin^2 A=\cos^2 A-\left(1-\cos^2 A\right)=2\cos^2 A-1$ OR $\cos 2A=\cos^2 A-\sin^2 A=\left(1-\sin^2 A\right)-\sin^2 A=1-2\sin^2 A$ (i) $\cos^2 A=\frac{\cos 2A+1}{2}$, (ii) $\sin^2 A=\frac{1-\cos 2A}{2}$ *	M1M1 A1A1cso [4]
(b)	$\left(2\sin x-\cos x\right)\left(\sin x-3\cos x\right)$ $=2\sin^2 x-7\sin x\cos x+3\cos^2 x$ $=2\left(\frac{1-\cos 2x}{2}\right),\ldots\ldots\ldots\ldots\ldots,+3\left(\frac{\cos 2x+1}{2}\right)$ $=\ldots\ldots\ldots\ldots-\frac{7\sin 2x}{2}+\ldots\ldots\ldots\ldots\ldots\ldots$ $=\frac{1}{2}\left(\cos 2x-7\sin 2x+5\right)*$	M1 M1, M1 M1 A1 cso [5]
(c)	$y=\frac{1}{2}\left(\cos 2x-7\sin 2x+5\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\sin 2x-7\cos 2x=0$ $\Rightarrow-\sin 2x=7\cos 2x\Rightarrow\tan 2x=-7$ $2x=-81.869,\quad 98.130,\quad 278.130$ $\Rightarrow x=49,\quad 139$	M1 M1 A1 A1 [4]
		Total 13 marks

Part	Mark	Notes
NB: Parts (a) and (b) are proofs with given answers. You must check every line of their working.		
(a)	**M1**	Uses the formula for $\cos(A+B)=\cos A\cos B-\sin A\sin B$ correctly.
	M1	Uses the identity $\cos^2 A=1-\sin^2 A$ or $\sin^2 A=1-\cos^2 A$
(i)	**A1 cso**	For the correct identity for either $\sin^2 A$ or $\cos^2 A$ with no errors seen
(ii)	**A1 cso**	For the correct identity for $\sin^2 A$ and $\cos^2 A$ with no errors seen
SC	If a candidate does not use $\cos(A+B)=\cos A\cos B-\sin A\sin B$ to find the required identities for $\sin^2 A$ or $\cos^2 A$ but uses them to show $\sin^2 A=\frac{1-\cos 2A}{2}$ or $\cos^2 A=\frac{\cos 2A+1}{2}$ apply the following scheme.	
	First M mark – M0	
	2nd M1	Uses the identity $\cos^2 A=1-\sin^2 A$ or $\sin^2 A=1-\cos^2 A$ For example:

(i)/(ii)		$\cos^2 A = \frac{\cos 2A+1}{2} = 1-\sin^2 A$ $\sin^2 A = 1-\frac{\cos 2A+1}{2} = \frac{2-(\cos 2A+1)}{2} = \frac{1-\cos 2A}{2}$
	A1 cso	For the correct identity for $\sin^2 A$ or $\cos^2 A$ with no errors seen
	A0	Not available
(b)	**M1**	Multiplies out the brackets with no errors.
	M1	Substitutes at least one of the given identities for cos2x into their expansion [either for $\sin^2 x$ or $\cos^2 x$]. This is not a dependent mark, allow for substitution into an erroneous expression.
	M1	Substitutes both of the given identities for cos2x into their expansion [for both $\sin^2 x$ and $\cos^2 x$]. This is not a dependent mark, allow for substitution into an erroneous expression. **NB: This is a B mark in Epen**
	M1	Uses the identity sin2A = 2sin$A$cos$A$ seen applied explicitly **before** the final answer. Look for, $k\sin x\cos x \to \frac{k\sin 2x}{2}$
	A1 cso	For the correct identity with no errors seen.
	ALT – Works backwards – these steps must contain no errors (to maintain compatibility with the main method. Starting from $\frac{1}{2}(\cos 2x - 7\sin 2x + 5)$ award as follows:	
	M1	Uses: $\cos 2x = \cos^2 x - \sin^2 x$
	M1	Uses: $5 = 5\cos^2 x + 5\sin^2 x$
	M1	Uses: $-7\sin 2x = 14\sin x\cos x$
	M1	Collects like terms and divides by 2 to give: $3\cos^2 x - 7\sin x\cos x + 2\sin^2 x$
	A1	Factorises their expression to give: $(2\sin x - \cos x)(\sin x - 3\cos x)$
(c)	**M1**	For an attempt to differentiate the given equation. A minimally acceptable is defined as follows: $\frac{dy}{dx} = \pm k\sin 2x \pm l\cos 2x$ where k and l are integers **with no other terms**.
	M1	Uses the identity $\tan 2x = \frac{\sin 2x}{\cos 2x}$ This is an independent mark, but it must be used correctly following correct division of $\cos 2x$ throughout.
	A1	Find at least one **correct** value for 2x e.g., –81.869, 98.130, 278.130 etc **NB: This is an M mark in Epen. [**Accept – 81, 98, 278]
	A1	For x = 49 and 139 rounded to the nearest whole number or better.

	ALT - Differentiates using product rule	
	M1	$y=(2\sin x-\cos x)(\sin x-3\cos x)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=(2\sin x-\cos x)(\cos x+3\sin x)+(\sin x-3\cos x)(2\cos x+\sin x)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin x\cos x+6\sin^2 x-\cos^2 x-3\sin x\cos x+$ $+2\sin x\cos x+\sin^2 x-6\cos^2 x-3\sin x\cos x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin x\cos x+6\sin^2 x-7\cos^2 x$ **Allow** $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm p\sin x\cos x\pm q\sin^2 x\pm r\cos^2 x$ **NB: This will not factorise**
	M1	Divides through by $\cos^2 x$ provided they have an expression of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm p\sin x\cos x\pm q\sin^2 x\pm r\cos^2 x$ This gives : $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin x\cos x+6\sin^2 x-7\cos^2 x=0 \quad \rightarrow \div\cos^2 x$ $-7-2\tan x+7\tan^2 x=0$ **AND** solves to give a value for $\tan x=\frac{1\mp 5\sqrt{2}}{7}$ or allow $\tan x=\frac{1\pm 5\sqrt{2}}{7}$
	A1	At least one correct value for x of for example, awrt 49.1….awrt – 40.9
	A1	For both 49° and 139°

Question	Scheme	Marks
10(i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \left[(3a+2)\mathbf{i} + b\mathbf{j}\right] - \left[(b+1)\mathbf{i} + b\mathbf{j}\right]$	M1
	$\overrightarrow{AB} = \left[(3a+2) - (b+1)\right]\mathbf{i} = 3\mathbf{i}$	A1
	$(3a+2) - (b+1) = 3 \Rightarrow 3a - b = 2$ or $b = 3a - 2$	M1A1
	$\dfrac{\sqrt{17}}{34} = \dfrac{1}{\sqrt{68}}$	B1
	$68 = (3a+2)^2 + b^2$	M1
	$\Rightarrow 68 = (3a+2)^2 + (3a-2)^2 \Rightarrow 68 = 18a^2 + 8$	M1
	$\Rightarrow 60 = 18a^2 \Rightarrow a = \dfrac{\sqrt{30}}{3}$ oe	A1
(ii)	$b = \dfrac{3\sqrt{30}}{3} - 2 = \sqrt{30} - 2$	M1A1 [10]
		Total 10 marks

Part	Mark	Notes
	M1	For the vector equation $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ This can be implied by the next correct step.
	A1	For the correct unsimplified vector $\overrightarrow{AB} = \left[(3a+2)\mathbf{i} + b\mathbf{j}\right] - \left[(b+1)\mathbf{i} + b\mathbf{j}\right]$
	M1	For setting their vector $\overrightarrow{AB} = 3\mathbf{i}$ and attempting to find an expression in terms of *a* and *b* only
	A1	For a correct expression in any form. e.g. $3a - b = 2$ or $b = 3a - 2$ or equivalent
	ALT	
	M1	For the vector equation $\overrightarrow{OB} = \overrightarrow{AB} + \overrightarrow{OA}$
	A1	For the correct unsimplified vector $\overrightarrow{OB} = \left[(b+1)\mathbf{i} + b\mathbf{j}\right] + 3\mathbf{i} = \left\{(b+4)\mathbf{i} + b\mathbf{j}\right\}$
	M1	For setting their $\overrightarrow{OB}$ equal to $(3a+2)\mathbf{i} + b\mathbf{j}$ and equating coefficients of **i** and **j** $(b+4)\mathbf{i} + b\mathbf{j} = (3a+2)\mathbf{i} + b\mathbf{j} \Rightarrow b + 4 = 3a + 2$
	A1	For a correct expression in any form.

(i)		e.g. $3a-b=2$ or $b=3a-2$ or equivalent
	B1	For simplifying $\frac{\sqrt{17}}{34}$ into $\frac{1}{\sqrt{68}}$ or any equivalent. For example, $\left(\frac{\sqrt{17}}{34}\right)^2=\frac{1}{68}$ Accept seen anywhere in their working. If this step is not explicitly, correct *a* or *b* implies correct work.
	Way 1	
	M1	For using Pythagoras to form an equation with their $\sqrt{68}$ or 68 and $(3a+2)\mathbf{i}+b\mathbf{j}$ and forming an expression in terms of *a* and *b* [These values must be applied correctly. Do not accept $\frac{1}{68}=(3a+2)^2+b^2$ for this mark]
	M1	For eliminating *b* from their equation **and** solving the resulting equation to find a value for *a* or a^2.
	A1	For $a=\frac{\sqrt{30}}{3}$ oe This does not have to be simplified.
(ii)	**M1**	For finding the value of *b* using their expression in terms of *a* and *b* and their *a* **provided their *a* > 0**
	A1	For *b* = $\sqrt{30}-2$ o.e. This does not have to be simplified.
(ii)	**Way 2**	
	M1	For using Pythagoras to form an equation with their and $\overrightarrow{OB}=(b+4)\mathbf{i}+b\mathbf{j}$ $\sqrt{68}$ or 68 and forming an expression in terms of *b* [This must be applied correctly. Do not accept $\frac{1}{68}=(b+4)^2+b^2$ for this mark]
	M1	For finding a value for *b* or b^2 by solving their 3TQ by any valid method. E.g., by completing the square. $68=(b+4)^2+b^2$ $\Rightarrow 68=2b^2+8b+16\Rightarrow b^2+4b-26=0$ $\Rightarrow b^2+4b-26=(b+2)^2-30=0$ $\Rightarrow b=\sqrt{30}-2$
	A1	For *b* = $\sqrt{30}-2$ o.e. This does not have to be simplified.
(i)	**M1**	For finding the value of a using their expression in terms of *a* and *b* and their *b* **provided their *b* > 0.**
	A1	For $a=\frac{\sqrt{30}}{3}$ oe This does not have to be simplified.

Question	Scheme	Marks
11(a)	$\mathrm{f}\left(x\right)=10+6x-x^2=-\left(x^2-6x\right)+10\Rightarrow A=-1$ $\mathrm{f}\left(x\right)=-\left[\left(x-3\right)^2-9\right]+10=-\left(x-3\right)^2+19$ $B=-3,\ \ C=19$	B1 M1 A1A1 [4]
(b)	(i) $x = 3$ (ii) $\mathrm{f}\ (x)_{\text{greatest}} = 19$	B1ft B1ft [2]
(c)	$x^2-x+13=10+6x-x^2\Rightarrow 2x^2-7x+3=0$ $\Rightarrow\left(2x-1\right)\left(x-3\right)=0\Rightarrow x=\frac{1}{2},\ \ 3$	M1 M1A1 [3]
(d)	$A=\int_{'\frac{1}{2}'}^{'3'}\left(10+6x-x^2\right)\,\mathrm{d}x-\int_{'\frac{1}{2}'}^{'3'}\left(x^2-x+13\right)\,\mathrm{d}x=\left[\int_{'\frac{1}{2}'}^{'3'}-2x^2+7x-3\ \mathrm{d}x\right]$ $A=\left[-\frac{2x^3}{3}+\frac{7x^2}{2}-3x\right]_{'\frac{1}{2}'}^{'3'}$ $A=\left[\left(-\frac{2\times3^3}{3}+\frac{7\times3^2}{2}-3\times3\right)-\left(-\frac{2\times0.5^3}{3}+\frac{7\times0.5^2}{2}-3\times0.5\right)\right]$ $A=\frac{125}{24}$ oe	M1 M1A1 M1 A1 [5]
		Total 14 marks

Part	Mark	Notes
(a)	**B1**	For factorising – 1 and finding *A* = – 1
	M1	For an attempt to complete the square – (See general guidance)
	A1	For either $B=-3$ or $C=19$
	A1	For both $B=-3$ and $C=19$ Accept all values embedded.
	NB: Correct values following no working – Award full marks in this part.	

(b)(i)	**B1ft**	$x = 3$ ft their *B* You may see differentiation to find a maximum *x*. Allow a correct value of *x* here even if it does not follow from their working.
(b)(ii)	**B1ft**	$\mathrm{f}(x) = 19$ ft their *C* You may see differentiation to find a maximum *x*. Allow a correct value of $\mathrm{f}(x)$ here even if it does not follow from their working. Allow $y = 19$.
(c)	**M1**	Sets the equation of curve *C* equal to the equation of curve *S* and forms a 3TQ
	M1	Attempts to solve their 3TQ using any correct method to find **two** values of *x* If their 3TQ is incorrect and no working is seen to solve it, withhold this mark.
	A1	For both $x = \frac{1}{2},\ 3$ correct Accept coordinates – ignore *y* values.
	NB: Both **c**orrect values following no working – Award full marks in this part.	
(d)	**M1**	For a correct statement with their values of *x* in the correct position of the intent to integrate the two expressions and **subtract** the result, or to integrate $2x^2 - 7x + 3$ Ft their limits for this mark, the correct way around, but allow recovery at a later stage. If they deal with the two expressions separately, look for subtraction at the end of their solution.
	M1	For an attempt to integrate their combined expression or **two** separate expressions. They must achieve as a minimum $\pm Px^3 \pm Qx^2 \pm Rx$ for their integration whether they integrate two expressions separately (each) or one combined expression. Ignore limits for this mark.
	A1	For the correct integrated expression(s)– ignore limits for this mark Allow $\frac{2x^3}{3} - \frac{7x^2}{2} + 3x$
	M1	For substituting their limits into their integrated expression and subtracting the results of lower limit from upper limit. **Note carefully:** If they have the correct limits and correct integration allow this mark if the **correct** area is obtained without seeing explicit substitution. $\left(\pm\frac{125}{24},\quad \pm 5.208...\right)$ If their limits are incorrect or the integration is incorrect or the final area is incorrect then explicit substitution **must** be seen for the award of this mark.
	A1	For the correct area $A = \frac{125}{24}$ oe

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